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JEE Main Code
PRAMOD VAKACHARLA
355 out of 360

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CONTENTS

- Maths Musing Problem Set - 138 4
- Cracking the JEE Main Code 6
- CBSE Board Chapterwise Series-1 8
- Brain Twisters 14
- You Asked, We Answered 17
- Problems from Team Selection Tests 19
- Enrich Your Concepts Sets - Class XI 32
- Solved Paper Kerala PET - 2014 39
- Solved Paper WB JEE - 2014 60
- Maths Musing - Solutions 76
- Solved Paper Karnataka CET - 2014 78

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Editor : Anil Ahlawat

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editorial

Mathematicians - Are they born Mathematicians or developed into Mathematicians?

If experimenters are to be believed, both may be right-born mathematicians, developed into mathematicians or by physical transformation. We have read about Kalidasa, born as a shepherd becoming a great poet overnight. An accident is normally not good for anybody. But here is a 'miracle'. Jason Padgett was an ordinary furniture salesman. He received sudden brain injuries when two persons assaulted him. Suddenly he became a genius in mathematics. He saw circles as overlapping triangles, and could draw complex geometrical shapes. He saw shapes when mathematical equations were shown.

Brain scientists now studied his brain by MRI scanning. It was observed that the parietal cortex appears to become more active and lit up the most, while areas involved with visual memory, sensory processing and planning also showed activity. These abilities were lying dormant in Padgett's brain but were activated by the injury.

This study also confirms the work done by the Oxford University scientists in determining the parts of the brain which get activated for different tasks. Indian scientists can combine these studies with the highly evolved yogis, psychology experts and the usefulness of other exercises to improve the intelligence. We do not advocate the path of injury! We can make use of simpler exercises to excite our brain.

Anil Ahlawat
Editor

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today with the suggestion of Shri Mahabir Singh. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

Prof. Dr. Ramanaiah Gundala, Former Dean of Science and Humanities, Anna University, Chennai

PROBLEM Set 138

JEE MAIN

1. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{1, 2, 3, ..., 12\}$. If M is the number of increasing functions from A to B and N is the number of onto functions from B to A which are non increasing, then the sum of the digits of $(M - N)$ is

(a) 12 (b) 13 (c) 14 (d) 15

2. $\int_0^{\infty} e^{-2x}(x^4 - 2x^3 + 4x^2 - 2x - 1)dx =$

(a) -2 (b) -1 (c) 0 (d) 1

3. The incircle of triangle ABC touches the sides BC, CA, AB at D, E, F. The ratio of the areas of triangles DEF and ABC is

(a) $\frac{r}{R}$ (b) $\frac{2r}{R}$ (c) $\frac{r}{2R}$ (d) $\frac{2r}{3R}$

4. Let $\{x, y\}$ be a subset of the set $\{1, 2, 3,..., 10\}$. The probability that $|x - y| \le 5$ is

(a) $\frac{2}{9}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{9}$

5. $\cot\left(-i\ln\left(\frac{x+iy}{x-iy}\right)\right) =$

(a) $\frac{x^2 - y^2}{2xy}$ (b) $\frac{2xy}{x^2 - y^2}$

(c) $\frac{2xy}{x^2 + y^2}$ (d) $\frac{y^2 - x^2}{2xy}$

JEE ADVANCED

6. A plane $ax + by + cz = 105$ contains the line $2x - y + z - 3 = 0 = 3x + y + z - 5$. The point $(2, 1, -1)$ is at a distance of $\frac{1}{\sqrt{6}}$ from the plane. Then $a + b + c =$

(a) 110 (b) 90 (c) 70 (d) 50

COMPREHENSION

Consider the curves $x^2 + y^2 = 2$ and $x^2 + 4y^2 = 4$

7. The area common to the two curves is

(a) $\tan^{-1} 1$ (b) $2\tan^{-1} 2$

(c) $4\tan^{-1}\sqrt{2}$ (d) $4\tan^{-1} 2\sqrt{2}$

8. The length of a common tangent is

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $\frac{1}{\sqrt{2}}$

INTEGER MATCH

9. If $z = i\ln(7 - 4\sqrt{3})$, then $\cos z$ is

MATCHING LIST

10. In triangle ABC, $r_1 = 2$, $r_2 = 1$ and $C = \frac{\pi}{2}$. Then,

List-I		List-II	
P.	$2a =$	1.	$\sqrt{17} - 3$
Q.	$2b =$	2.	$\sqrt{17} - 1$
R.	$2r_3 =$	3.	$\sqrt{17} + 1$
S.	$2r =$	4.	$\sqrt{17} + 3$

	P	Q	R	S
(a)	1	3	2	4
(b)	4	1	3	2
(c)	2	4	1	3
(d)	3	2	4	1

Prof. Ramanaiah is the author of MTG JEE(Main & Advanced) Mathematics series

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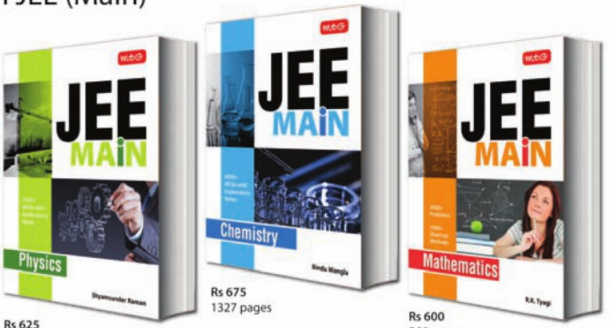

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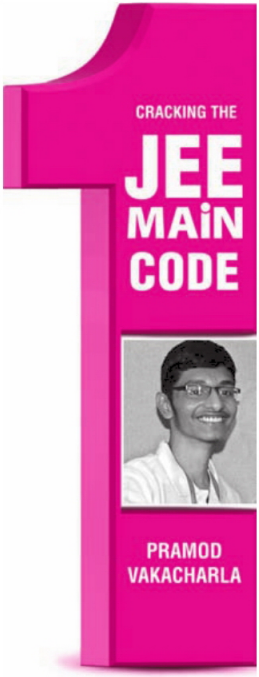
JEE Main 2014 Topper Interview : How Pramod Vakacharla scored 355 out of 360?

JEE Main 2014 highest scorer Pramod Vakacharla got 355 marks out of total 360 and outsmarted over 13 lakh candidates who took JEE Main 2014. Pramod says he missed the perfect score of 360 as he marked a wrong bubble for a Physics question in the OMR Answer sheet of JEE Main. He, however, is not taking any chance for JEE Advanced 2014 scheduled for May 25.



Congratulations for your outstanding performance in JEE Main 2014! How did you make it possible?

Thank you. I could do it with my serious preparations and support from my teachers and parents.

What was your overall and section scores in JEE Main 2014?

My overall score is 355. I scored 120 in both Mathematics and Chemistry, and my Physics score was 115.

That is indeed a great achievement! This means that you made only one incorrect answer out of 90 questions in JEE Main paper.

Yes. Fortunately, I could do all questions in the paper except one question in Physics. Actually, I knew the right answer to that Physics question also, but marked the wrong bubble in the OMR Answer sheet.

Your performance in IIT JEE brings you the title of 'perfect scorer'. Did you expect this high score before your exam on April 6?

Actually, I had been scoring above 320-330 in all my practice tests before JEE Main. So, I was expecting a similar score in the JEE Main result also. But, I was not sure that I would score highest in the JEE Main 2014.

Now that you are the topper of JEE Main, what do you think was the key factor behind your spectacular performance in this most competitive Engineering exam?

My focus was never on the marks. I always dedicated my time and energy on solving 100 per cent questions in my test series. But, I never expected that I will do almost all questions correctly in the actual test.

What was your preparation strategy and routine study period for JEE Main?

I dedicated around 11 hours daily on studies. I spent 4 hours on my daily classes and rest 7 hours on self study. Overall, I divided the daily preparations into three parts – 3 hours each for Physics and Mathematics and 4 hours for Chemistry. Since I put up in the hostel of Narayana Sri Chaitanya IIT Academy, Vijaywada, I consult my teachers whenever I have doubts on any question or any topic. I naturally go and discuss with them random problems related to my preparations.

Which subject was easiest and which was the toughest?

Since my favorite subject is Chemistry, I found the questions relatively easier. Physics was a bit difficult and Mathematics was of average difficulty level.

So, it was quite a hectic preparation schedule for you. Could you find time to connect with your friends on Facebook during your preparation?

No. I could not get any free time to chat online with my friends during my preparation. I lost logged in to my Facebook page some 6 months back. But, some time I watch movies that are either Telugu or a Hollywood flick. Sometimes, I also play Badminton which is my favorite sports.

Which is your favourite Hollywood movie?

…Hmm… I like many Hollywood movies. But, my favourite is 'Fast and Furious'. I have watched all 6 parts of this movie and eagerly waiting to watch the next sequel (The 7th installment of upcoming 2015 American action film).

So, once you crack JEE Advanced, which Engineering branch would you like to opt and which IIT will you prefer?

Though, I am yet to get into JEE-Advanced rankings, but I would prefer Computer Science Engineering (CSE) in IIT Bombay.

How do you plan your preparation for JEE Advanced?

I have already begun my preparations and will be maintaining my study schedule as previously. I will try to put in an extra hour or so for more practice in Physics. My test series for JEE Advanced are on and I am fully concentrating on doing well in JEE Advanced 2014.

Courtesy : Careers360.com

- The cut-off for the entrance test to the Indian Institutes of Technology is a tad higher this year, at 115 for general category candidates compared to 113 last year. For OBC candidates, it stands at 74, compared to 70 in 2013, at 53 for scheduled caste students and 47 for scheduled tribe candidates.
- Total 1.54 lakh candidates, 28,666 of them girls are selected for JEE Advanced.

STATE TOPPERS

		Score
Tamil Nadu State Topper	Ravi Teja A.V.	340/360
Punjab State Topper	Saksham Sharma	337/360
Chattisgarh State Topper	C. Vishwesh	335/360
Haryana State Topper	Divpreet	331/360
West Bengal State Topper	Gyanesh Gupta	329/360
Karnataka State Topper	Saurabh Pinjani	325/360

ADMISSIONS & A NEW TEST

EXAMINEES

2013	**2014**
13,50,000	**13,57,000**

JEE (Main) RESULTS

- JEE (Main) results are available on http://cbseresults.nic.in/jee/jee_cbse_2014.htm
- The JEE (Main) paper had 90 questions carrying a total of 360 marks.
- CBSE released the answer keys on April 28 and candidates to submit their feedback within a day. Only candidates were allowed to challenge the answers, that too for a fee of 1,000.

JEE (Advanced) RESULTS

- Candidates in the top 1.5 lakh rank list can take the JEE (Advanced) on May 25. Admission to the 13 IITs and ISM Dhanbad will be based on JEE (Advanced) scores.
- Nearly 15,500 seats are on offer in 30 NITs, 850 seats in five IIITs and another 15,000 seats in self-financing institutions. The rank scored in the JEE (Main) is valid for two years. It means a candidate who figures in the top 1.5 lakh list of JEE (Main) can take the JEE (Advanced) either this year or next year.
- Last year, to figure in the top-20 percentile list, the cut-off was the highest for Andhra Pradesh—91.8%, followed by Tamil Nadu at 90.9% and Kerala at 85.2%.

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Series-1

Relations and Functions

RELATIONS

Definition : Let A and B be two non-empty sets, then a relation R from A to B is a subset of $A \times B$. The set of all first elements of the ordered pairs in a relation R from set A to set B is called domain of the relation R. The set of all second elements in a relation R from set A to set B is called range of the relation R. The whole set B is called the co-domain of the relation R. Also, range is always a subset of co-domain.

NUMBER OF RELATIONS

Let X and Y be two non-empty finite sets having m and n elements respectively, then number of ordered pairs in $X \times Y$ is $m \times n$.

So, the number of subsets of $X \times Y = 2^{m \times n}$

i.e. Number of relations from X to $Y = 2^{mn}$

For example, if $X = \{4, 5, 6\}$, $Y = \{7, 8\}$, then number of relations from X to Y

= Number of subsets of $X \times Y = 2^6 = 64$.

- **Inverse Relation :** If R is a relation on A, then the relation R^{-1} on A, defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$

❑ VARIOUS TYPES OF RELATIONS

- **Empty Relation :** A relation R in a set A is called empty relaion, if no element of A is related to any element of A, i.e., $R = \phi \subset A \times A$.
- **Universal Relation :** A relation R in a set A is called universal relation, if each element of A is related to every element of A, *i.e.*, $R = A \times A$.
- **Reflexive Relation :** If $a\ R\ a\ \forall\ a \in A$ *i.e.* if $(a, a) \in R\ \forall\ a \in A$.
- **Symmetric Relation :** A relation R on a set A is said to be symmetric, iff $aRb \Rightarrow bRa$ *i.e.*, if $(a, b) \in R \Rightarrow (b, a) \in R$. It is easy to see that a relation R on a set is symmetric iff $R^{-1} = R$.
- **Transitive Relation :** If aRb and $bRc \Rightarrow aRc$ *i.e.*, if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.
- **Antisymmetric Relation :** Let A be any set. A relation on set A is said to be an antisymmetric relation iff $(a, b) \in R$ and $(b, a) \in R$ $\Rightarrow a = b\ \forall\ a, b \in A$.

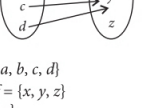
  Note : The identity relation on a set A is an antisymmetric relation.
- **Equivalence Relation :** A relation which is reflexive, symmetric and transitive, is called an equivalence relation.

FUNCTIONS

Let A and B be two non-empty sets. A function f is defined from A to B, denoted by $f : A \to B$, if all the elements of set A have a unique image in set B. Here A is called the domain and B is called the co-domain of the function.

The set of all those elements of set B that have at least one pre-image in set A, called the range of f.

e.g., Let $A = \{a, b, c, d\}$, $B = \{x, y, z\}$ let $f : A \to B$ be defined as :



then,

Domain of $f = \{a, b, c, d\}$

Co-domain of $f = \{x, y, z\}$

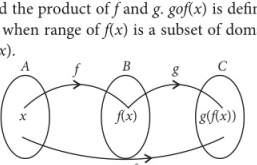
Range of $f = \{x, y\}$

TYPES OF FUNCTIONS

- **One-one function :** A function $f : A \to B$ is said

to be one-one function or injective function if different elements in A have different images in B.
Thus, $f: A \to B$ is one-one function if $x_1, x_2 \in A$, $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$
or if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

- **Many-one function :** A function $f: A \to B$ is said to be many-one function if two or more than two elements in A have the same image in B.
e.g., Let $A = \{-1, 1, 2, 3\}$ and $B = \{1, 4, 9\}$ Consider $f: A \to B: f(x) = x^2$ then each element of A has a unique image in B, but two elements -1 & 1 of A have same image $1 \in B$, so f is a many-one function.
- **Onto or Surjective function :** A function $f: A \to B$ is said to be an onto or surjective function if every element in B has atleast one pre-image in A. Thus if f is onto, then corresponding to each $y \in B$ there is atleast one element $x \in A$ such that $y = f(x)$. Hence f is onto $\Leftrightarrow$ range of $f = B$.
- **Into function :** A function $f: A \to B$ is said to be an into function if there exists even a single element in B having no pre-image in A.
- **Bijective function :** A function $f: A \to B$ is said to be a bijective function, if it is both one-one and onto function.
e.g. Let $A = \{x, y, z\}$, $B = \{a, b, c\}$ consider $f: A \to B$ such that $f(x) = a$, $f(y) = b$ and $f(z) = c$ then f is both one-one and onto.
- **Constant function :** Let $f: A \to B$. f is defined such that all the elements in A always have the same image in B, then f is said to be constant function.
e.g. Let $A = \{1, 2, 3\}$, $B = \{5, 7, 9\}$. Consider $f: A \to B, f(x) = 5 \ \forall \ x \in A$, hence f is a constant function.
- **Identity function :** Let A be a non-empty set, then the identity function (denoted by I_A) defined by $I_A : A \to A : I_A(x) = x, \ \forall \ x \in A$. Hence identity function maps each x into itself and it is a one-one onto function with domain A and range A.
- **Equal function :** Functions f and g are said to be equal, if they have same domain and satisfy the condition $f(x) = g(x), \ \forall \ x$.
- **Even and odd function :** A function $f: A \to B$ is said to be an even or odd function according as $f(-x) = f(x) \ \forall \ x \in A$ and $f(-x) = -f(x) \ \forall \ x \in A$ respectively.
e.g. $f: R \to R, f(x) = x^2$ is an even function since $f(-x) = f(x) \ \forall \ x \in R$ and $f(x) = x^3$ is an odd function since $f(-x) = -f(x) \ \forall \ x \in R$.
- **Inverse function :** Let f be a one-one onto function from A to B. Let y be an arbitrary element of B, then f being onto, there exists an element $x \in A$ such that $f(x) = y$ (x must be unique).
Thus for each $y \in B$ there exist a unique element $x \in A$ such that $f(x) = y$. So we may define a function defined by $f^{-1} : B \to A : f^{-1}(y) = x \Leftrightarrow f(x) = y$.
Hence f^{-1} is called the inverse of f. Hence a function f is invertible if and only if f is one-one onto.
Note : Let A and B be two non-empty sets. Let f be a one-one onto from A to B, then f^{-1} is also one-one and onto.
- **Composite function :** Let A, B and C be three non-empty sets. Let $f: A \to B$ and $g: B \to C$, then the composite function gof is defined from $A \to C : gof(x) = g(f(x))$. Composite function is called the product of f and g. $gof(x)$ is defined only when range of $f(x)$ is a subset of domain of $g(x)$.



Note :
(i) The product of any function with the identity function is the function itself.
(ii) The product of any invertible function f with its inverse function f^{-1}, is an identity function.

(iii) Composite function is associative.

(iv) Let $f : A \to B$ and $g : B \to A$ such that gof is an identity function on A and fog is an identity function on B, then $g = f^{-1}$.

(v) If $f : A \to B, g : B \to C$, then $fog \neq gof$

(vi) Let $f : A \to B$ and $g : B \to C$ be one-one onto function, then gof is also one-one onto and $(gof)^{-1} = f^{-1} og^{-1}$.

BINARY OPERATIONS

A binary operation '$*$' on A is a rule that associates with every ordered pair (a, b) of $A \times A$, to a unique element of A. When $* : A \times A \to A$ is well defined, then it is customary to say that the set A is closed under the binary operation '$*$'.

Let A be a non-empty set and '$*$' be a binary operation on A, then

1. A is said to be closed under the operation '$*$'.
2. The operation is said to be commutative iff $a * b = b * a$ for all $a, b \in A$.
3. The operation is said to be associative iff $(a * b) * c = a * (b * c)$ for all $a, b, c \in A$.
4. The operation is said to be
   - **(i)** Left cancellative iff $a * b = a * c$ implies $b = c$ for all $a, b, c \in A$.
   - **(ii)** Right cancellative iff $b * a = c * a$ implies $b = c$ for all $a, b, c \in A$.
   - **(iii)** cancellative if it is left as well as right cancellative.
5. An element $e \in A$ is said to be an identity element iff $e * a = a = a * e$ for all $a \in A$.
6. An element $a \in A$ is called invertible iff there exists an element $b \in A$ such that $a * b = e = b * a$, b is called inverse of a and is written as a^{-1}.

   Remark : If '$*$' is a binary operation on A, then the pair $(A, *)$ stands for the set A which is closed under the operation '$*$'.

   If $*$ is associative on A and $a, b, c \in A$, then either of $a * (b * c)$ and $(a * b) * c$ is written as $a * b * c$.

VERY SHORT ANSWER TYPE (1 MARK)

1. Show that the relation '$>$' on the set R of all real numbers is transitive but it is neither reflexive nor symmetric.
2. Let $A = \{a, b, c\}$, $B = \{x, y, z\}$, a rule is given by $f(a) = x, f(b) = y, f(b) = z, f(c) = z$. Is f a function from A to B?
3. If $f : N \to N$ is defined by $f(n) = n^2$, then show that f is a function, also find its range.
4. Let $f(x) = \dfrac{x+1}{x-1}$. Show that
   $$f(x) + f\left(\frac{1}{x}\right) = 0 \text{ for } x \neq 1.$$
5. Let $f : R \to R : f(x) = \sin x$ and $g : R \to R : g(x) = x^2$, prove that $gof \neq fog$.
6. $f : R \to R$ defined by $f(x) = x^2 + 1$, then show that $f(x)$ is not one-one function.

SHORT ANSWER TYPE (4 MARKS)

7. Show that relation 'is congruent to', on the set of all triangles in a plane is an equivalence relation.
8. Find domain and range of the following functions:
   (a) $\dfrac{x}{1+|x|}$ (b) $\dfrac{1}{3x+2}$
9. Let $f : R \to R : f(x) = 3x - 4$. Show that f is an invertible function also find f^{-1}.
10. Let $f(x) = x + 2$ and f has domain $\{x : 2 \le x \le 5\}$. Find f^{-1} and its domain and range.
11. If the function f and g are given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, find fog and gof.
12. Let a binary composition 'o' be defined on Z by $lom = l - m + lm$ for all $l, m \in Z$. Prove that the composition 'o' is neither commutative nor associative.

LONG ANSWER TYPE (6 MARKS)

13. Discuss the commutativity and associativity of the binary operation $*$ on R defined by
    $$a * b = \frac{ab}{4} \text{ for all } a, b \in R.$$
14. Let $A = N \times N$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that $*$ is commutative and associative. Find the identity element for $*$ on A, if any.

15. Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

SOLUTIONS

1. The relation '>' on R is transitive, since $a > b$ and $b > c \Rightarrow a > c$ so it is transitive, since no real number can be greater than itself, so the relation is not reflexive.

Also $a > b \Rightarrow b > a$. So it is not symmetric.

2. No, an element $b \in A$ is assigned to y and z, *i.e.*, more than one element of B. Therefore f is not a function.

3. For each natural number n is assigned to its square which is unique, hence $f(n)$ is a function and range is $(n^2 : n \in N)$.

4. We have, $f(x) = \dfrac{x+1}{x-1}$

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}+1}{\frac{1}{x}-1} = -\left(\frac{x+1}{x-1}\right) = -f(x)$$

$$\therefore \quad f(x) + f\left(\frac{1}{x}\right) = 0$$

5. $gof(x) = g(f(x)) = g(\sin x) = (\sin x)^2 = \sin^2 x$

$fog(x) = f(g(x)) = f(x^2) = \sin x^2$

Hence $gof \neq fog$.

6. For one-one function, if $x_1 \neq x_2$,

then $f(x_1) \neq f(x_2)$

So, $-1 \neq 1$, but $f(-1) = f(1)$

Hence $f(x)$ is not one-one function.

7. Let S be the set of all triangles in a plane then the congruence relation on S is

(i) Reflexive, since $\Delta \equiv \Delta$ for all $\Delta \in S$

(ii) Symmetric, since $\Delta_1 \equiv \Delta_2 \Rightarrow \Delta_2 \equiv \Delta_1$

(iii) Transitive, since $\Delta_1 \equiv \Delta_2$ and $\Delta_2 \equiv \Delta_3$

$\Rightarrow \Delta_1 \equiv \Delta_3$

Hence, above relation on S is an equivalence relation.

8. (a) $f(x) = \dfrac{x}{1+|x|}$, $f(x)$ is defined $\forall\ x \in R$, hence domain is R.

When $x \to \infty$ then $f(x) \to 1$ and when $x \to -\infty$ then $f(x) \to -1$.

Hence range of $f(x)$ is $(-1, 1)$.

(b) $f(x) = \dfrac{1}{3x+2}$ is undefined when $x = -\dfrac{2}{3}$, hence domain of $f(x)$ is $R - \left\{-\dfrac{2}{3}\right\}$. For range let $y = \dfrac{1}{3x+2} \Rightarrow x = \dfrac{1}{3}\left(\dfrac{1}{y} - 2\right)$.

Here $y \neq 0$, so range of $f(x)$ is $R - \{0\}$.

9. Domain of $f(x)$ is R, range is R, so $f(x)$ is bijective function, hence f^{-1} exists.

$y = f(x) = 3x - 4$

$$x = \frac{y+4}{3} \Rightarrow x = f^{-1}(y)$$

$$\therefore \quad f^{-1}(y) = \frac{1}{3}(y+4)\ \forall y \in R$$

$$\Rightarrow \quad f^{-1}(x) = \frac{1}{3}(x+4)\ \forall x \in R$$

10. Domain of $f = \{x : 2 \le x \le 5\}$

Range of $f = \{x + 2 : 2 \le x \le 5\}$

$= \{x + 2 : 4 \le x + 2 \le 7\}$

$= \{y : 4 \le y \le 7\}$

Hence range of $f(x)$ is $[4, 7]$

$x = y - 2 \Rightarrow x = f^{-1}(y)$

$\Rightarrow \quad f^{-1}(y) = y - 2 \Rightarrow f^{-1}(x) = x - 2$

$\therefore$ Domain of f^{-1} is $\{y : 4 \le y \le 7\}$,

Range of f^{-1} is $\{x : 2 \le x \le 5\}$.

11. Range of $g = \{1, 3\}$ and Domain of $f = \{1, 3, 4\}$

$\therefore$ Range of $g \subseteq$ domain of f and therefore fog is defined whose domain is the same as that of g which is $\{2, 5, 1\}$

$(fog)(2) = f(g(2)) = f(3) = 5$

$(fog)(5) = f(g(5)) = f(1) = 2$

$(fog)(1) = f(g(1)) = f(3) = 5$

$\Rightarrow fog = \{(2, 5), (5, 2), (1, 5)\}$

Again range of $f = \{1, 2, 5\}$ and domain of $g = \{1, 2, 5\}$

$\therefore$ Range of $f \subseteq$ domain of g and therefore gof is defined whose domain is the same as that of f, which is $\{1, 3, 4\}$

$(gof)(1) = g(f(1)) = g(2) = 3$

$(gof)(3) = g(f(3)) = g(5) = 1$

$(gof)(4) = g(f(4)) = g(1) = 3$

$\Rightarrow gof = \{(1, 3), (3, 1), (4, 3)\}$

12. **Commutativity :** As for $l, m \in Z$, $lom = l - m + lm$ and

$mol = m - l + ml$, therefore, $lom \neq mol$

$\Rightarrow$ 'o' is not commutative.

Associativity :

Also, for all $l, m, n \in Z$

$(lom)on = (l - m + lm)on$

$= (l - m + lm) - n + (l - m + lm)n$

$= l - m - n + lm + ln - mn + lmn$

and $lo(mon) = lo(m - n + mn) = l - (m - n + mn) + l(m - n + mn)$

$= l - m + n - mn + lm - ln + lmn$

So, $(lom)on \neq lo(mon)$.

$\Rightarrow$ 'o' is not associative.

Hence, composition 'o' is neither commutative nor associative.

13. We have, $a * b = \frac{ab}{4}$ for all $a, b \in R$.

Commutativity: For any $a, b \in R$, we have

$$a * b = \frac{ab}{4} \text{ and } b * a = \frac{ba}{4}$$

We know that the multiplication on R is a commutative binary operation.

$\therefore$ $ab = ba$ for all $a, b \in R$

$$\Rightarrow \frac{ab}{4} = \frac{ba}{4} \text{ for all } a, b \in R$$

$\Rightarrow$ $a * b = b * a$ for all $a, b \in R$.

So, '$*$' is a commutative binary operation on R.

Associativity: Let $a, b, c \in R$. Then,

$$(a * b) * c = \left(\frac{ab}{4}\right) * c = \frac{\left(\frac{ab}{4}\right)c}{4} = \frac{(ab)c}{16} \quad \ldots(i)$$

and, $$a * (b * c) = a * \left(\frac{bc}{4}\right) = \frac{a\left(\frac{bc}{4}\right)}{4} = \frac{a(bc)}{16} \quad \ldots(ii)$$

Since multiplication is an associative binary operation on R.

$\therefore$ $(ab)c = a(bc)$

$$\Rightarrow \frac{(ab)c}{16} = \frac{a(bc)}{16} \qquad \text{[Using (i) and (ii)]}$$

$\Rightarrow$ $(a * b) * c = a * (b * c)$ for all $a, b, c \in R$.

So, $*$ is an associative binary operation on R.

14. Let $(a, b), (c, d), (e, f) \in A = N \times N$ be arbitrary, then

$(a, b) * (c, d) = (a + c, b + d) = (c + a, d + b)$

$= (c, d) * (a, b)$

and $((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f)$

$= ((a + c) + e, (b + d) + f)$

$= (a + (c + e), b + (d + f))$

$= (a, b) * (c + e, d + f)$

$= (a, b) * ((c, d) * (e, f))$

$\therefore$ '$*$' is commutative and associative.

Let (p, q) be the identity element, then

$(a, b) * (p, q) = (p, q) * (a, b) = (a, b)$ for all $(a, b) \in A$

$\Rightarrow$ $(a + p, b + q) = (p + a, q + b) = (a, b)$ for all $(a, b) \in A$

$\Rightarrow$ $a + p = a$ and $b + q = b$ for all $(a, b) \in A$

$\Rightarrow$ $p = 0, q = 0$

$\therefore$ $(0, 0)$ is the identity element for '$*$'.

15. Let $x_1, x_2 \in D_f = R_+ = [0, \infty)$ be such that

$f(x_1) = f(x_2)$

$\Rightarrow x_1^2 + 4 = x_2^2 + 4 \Rightarrow x_1^2 = x_2^2 \Rightarrow |x_1| = |x_2|$

$\Rightarrow$ $x_1 = x_2$ $\quad (\because$ both $x_1, x_2 \geq 0)$

$\therefore$ f is one-one.

Let $y \in R_f$, then $y = f(x)$, $x \in D_f = R_+$

$\Rightarrow$ $y = x^2 + 4$

$\Rightarrow$ $x = \sqrt{y - 4}$ $\quad (\because$ $x \geq 0)$

As x is real, therefore, $y - 4 \geq 0$

$\Rightarrow$ $y \geq 4 \Rightarrow R_f = [4, \infty)$.

$\therefore$ $R_f =$ co-domain $\Rightarrow f$ is onto.

Also, corresponding to every $y \in R_f$ there exists $\sqrt{y-4} \in D_f$ such that $f(\sqrt{y-4}) = y$.

$\therefore$ f is both one-one and onto and hence invertible.

To find f^{-1} : Let $y = f(x)$

$\Rightarrow$ $y = x^2 + 4$

$\Rightarrow x^2 = y - 4 \Rightarrow x = \sqrt{y-4}$ $\quad (\because$ $x \geq 0)$

$\Rightarrow f^{-1}(y) = \sqrt{y-4}$ or $f^{-1}(x) = \sqrt{x-4}$

for all $x \in D_{f^{-1}} = R_f = [4, \infty)$

■■

mtG

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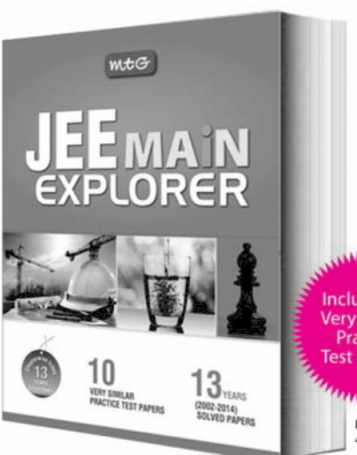

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Brain Twisters

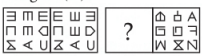
1. Select the figure from amongst the options which will continue the same series as established by the five problem figures.

Problem Figures

(a) (b)

(c) (d)

2. There is a definite relationship between figures A and B. Establish a similar relationship between figures C and D by selecting a suitable figure from the options that would replace the question mark (?) in figure (D).



A B C D

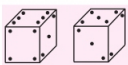
(a) (b)

(c) (d)

3. A cube is coloured red on one face, green on the opposite face, yellow on another face and blue on a face adjacent to the yellow face. The other two faces are left uncoloured. It is then cut into 125 smaller cubes of equal size. How many cubes are uncoloured on all the faces?

(a) 64 (b) 48 (c) 36 (d) 27

4. Two positions of a dice are shown below. When 2 dots is at the bottom, how many dots will be at the top ?



(i) (ii)

(a) 1 (b) 4

(c) 6

(d) can't be determined

5. Choose the option which most closely resembles the water-image of the given combination.

NhRqSy

(a) (b)

(c) (d)

Direction (6 and 7) : Read the following information to answer the questions.

Consider a group comprising of 4 students – Reena, Beena, Meena and Neena, who stand in a row. Reena and Beena stand in sixth and seventh positions respectively from the left. Meena and Neena stand in the fourth and fifth positions respectively from the right. When Beena and Meena exchange their positions, then Beena will be fifteenth from the left.

6. Originally, Neena's position from the left is

(a) 5th (b) 13th (c) 14th (d) 16th

7. If Neena and Reena also exchange their positions between themselves, then after the exchange, Neena's position from the left will be

(a) 6th (b) 10th

(c) 11th (d) none of these

Direction (8 and 9) : Read the following information to answer the questions.

B, D, H, K, P, R and T are seven friends who studied in three management institutes X, Y and Z. They had opted for one out of the three specializations *i.e.,* Marketing, Finance and Personnel with at least two of them in each specialization. No two persons have the same combination of institute and specialization.

The Marketing specialist from institute Y earns the maximum. D studied in institute Z with Personnel specialization and earns more than only B and K.

H studied in institute X with Marketing and earns less than only P and T. R studied in institute Z with Finance and earns more than D but less than H. T studied in institute X with Finance specialization. K, a Personnel specialist, studied in institute Y and is not the least earner among them. B is the only Finance specialist from one of these institutes among them.

8. Which of the following combinations of institute-individual-specialization is not correct?
(a) X-K-Personnel (b) X-H-Marketing
(c) Y-B-Finance (d) Z-R-Finance

9. How many of them earn more than D?
(a) One (b) Two
(c) Three (d) none of these

10. A code language has been used to write the words in capital letters in English in Column I as Greek letters in Column II. Greek letters in Column II do not appear in the same order as letters in Column I.

Column I	Column II
CLEAR	γβωπθ
VIEW	νεγδ
TURN	ηρπσ
BUTTER	σρασπγ
OILY	δλθμ
WRITE	γπσνδ
VOWEL	νλεγθ

Decode the language and choose the correct code for the word

LIVER

(a) νηλμπ (b) δγθπε (c) ρσωεν (d) αβδγη

11. Read the following information carefully and answer the question given below.

In a village of Bastar district in Madhya Pradesh, only two types of people live who belong to a tribal class. The first type is known as class A, while the other is known as class B. In that village, there is no other type of person except these two. The activities of both types of people are governed by perfectly patterned norms of social behaviour. Each person of the tribe has to obey the norms. They are rigid about this.

As far as marriage is concerned, the following norms are to be followed :

(A) The people of class A cannot marry any other member of their own class, though they can marry member of class B.
(B) After being married, each male member ceases to be a member of that class in which he was born but automatically, he becomes the member of the other class to which his wife belongs.
(C) As far as females are concerned, they remain the members of their own class after being married.
(D) On his birth, the child automatically becomes the member of his mother's class.
(E) When any male member becomes widower or divorcee, then he again belongs to the group in which he was born.
(F) Nobody can marry more than one person according to social laws.

Any class B female can have
(P) Grandfather born in class A
(Q) Grandmother born in class A
(a) Only (P) can be true
(b) Only (Q) can be true
(c) Either (P) or (Q) can be true
(d) Neither (P) nor (Q) can be true

12. A child is looking for his father. He went 90 metres in the East before turning to his right. He went 20 metres before turning to his right again to look for his father at his uncle's place 30 metres from this point. His father was not there. From here he went 100 metres to the North before meeting his father in a street. How far did the son meet his father from the starting point?
(a) 80 metres (b) 100 metres
(c) 140 metres (d) 260 metres

13. Read the following information carefully to answer the given question.

A word and number arrangement machine when given an input line of words and numbers, rearranges them following a particular rule in each step. The following is an illustration of input and rearrangement.

Input :	goal	63	57	home	five		task	82	17
Step I :	82	goal	63	57	home	five		task	17
Step II :	82	five	goal	63	57	home	task		17
Step III :	82	five	63	goal	57	home	task		17
Step IV :	82	five	63	goal	57	home	17		task

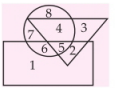
And Step IV is the last step.

As per the rules followed in the above steps, find out the appropriate step for the given intput.

Input : host 15 32 page 43 over mother 92 Which of the following steps will be the last but one?

(a) IV (b) V (c) VI (D) VII

14. In the given diagram, the triangle represents doctors, the circle represents players and the rectangle represents artists.



Which numbered space in the diagram represents doctors who are also players and artists?

(a) 1 (b) 3 (c) 4 (d) 5

15. In the given question, jumbled letters of a meaningful word are given. You are to rearrange these letters and select from the given options the word which is almost similar in meaning to the rearranged word.

I T G N D L E I

(a) Intelligent (b) Difficult
(c) Laborious (d) Quick

16. By applying which of the following meanings of arithmetical signs, will the value of $700 - 10 \div \frac{1}{2} \times 35 + 70$ be zero?

(a) × means ÷ , + means ×, ÷ means +, – means –
(b) × means ÷ , + means –, ÷ means ×, – means +
(c) × means + , + means –, ÷ means ×, – means ÷
(d) × means ÷ , + means –, ÷ means ×, – means –

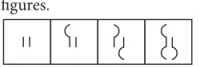
17. One night, three naughty boys stole a basketful of apples from the garden, hid the loot and went to sleep. Before retiring they did some quick counting and found that the fruits were less than a hundred in number. During the night one boy awoke, counted the apples and found that he could divide the apples into three equal parts if he first took one for himself. He then took one apple, ate it up and took $\frac{1}{3}$ of the rest, hid them separately and went back to sleep. Shortly thereafter, another boy awoke, counted the apples and he again found that if he took one for himself the loot could be divided into three equal parts. He ate up one apple, bagged $\frac{1}{3}$ of the remainder, hid them separately and went back to sleep. The third boy also awoke after sometime, did the same and went back to sleep. In the morning, when all woke up, and counted apples, they found that the remaining apples again totaled 1 more than could be divided into three equal parts. How many apples did the boy steal?

(a) 67 (b) 79
(c) 85 (d) none of these

18. A set of figures carrying certain characters is given. Assuming that the characters in each set follow a similar pattern, find the missing character.

(a) 35 (b) 48 (c) 72 (d) 120

19. Select a figure from amongst the options which will continue the series established by the four problem figures.



(a)
(b)
(c)
(d)

20. Which of the following figures (a), (b), (c) and (d), when folded along the lines, will produce the given Fig. (*X*)?

Fig. (*X*)

(a)
(b)
(c)
(d)

ANSWERS KEYS

1. (a)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (d)
6. (c)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (b)
11. (b)	**12.** (b)	**13.** (a)	**14.** (d)	**15.** (c)
16. (c)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (a)

YOU ASKED WE ANSWERED

Do you have a question that you just can't get answered?

Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

Q.1 Using mathematical induction, show that for all $n \in N$,

$$\tan^{-1}\left(\frac{x}{1+1\cdot 2\cdot x^2}\right)+\tan^{-1}\left(\frac{x}{1+2\cdot 3\cdot x^2}\right)+\ldots\ldots$$
$$+\tan^{-1}\left(\frac{x}{1+n(n+1)x^2}\right)=\tan^{-1}(n+1)x-\tan^{-1}x.$$

– Rajesh Thakur, Patna

Ans. Let $P(n)$:

$$\tan^{-1}\left(\frac{x}{1+1\cdot 2\cdot x^2}\right)+\tan^{-1}\left(\frac{x}{1+2\cdot 3\cdot x^2}\right)+\ldots\ldots$$
$$+\tan^{-1}\left(\frac{x}{1+n(n+1)x^2}\right)=\tan^{-1}(n+1)x-\tan^{-1}x.$$

For $n = 1$, we have

$$\text{L.H.S.} = \tan^{-1}\left(\frac{x}{1+1\cdot 2\cdot x^2}\right)=\tan^{-1}\left(\frac{2x-x}{1+2x^2}\right)$$
$$= \tan^{-1}2x-\tan^{-1}x = \text{R.H.S.}$$

Hence, the result is true for $n = 1$.

Assume that the result is true for $n = k$, *i.e.*,

$$P(k):\tan^{-1}\left(\frac{x}{1+1\cdot 2\cdot x^2}\right)+\tan^{-1}\left(\frac{x}{1+2\cdot 3\cdot x^2}\right)+\ldots.$$
$$\ldots\ldots+\tan^{-1}\left(\frac{x}{1+k(k+1)x^2}\right)$$
$$= \tan^{-1}(k+1)x-\tan^{-1}x \qquad \ldots(i)$$

For $n = k + 1$, we have

$$P(k+1):\tan^{-1}\left(\frac{x}{1+1\cdot 2\cdot x^2}\right)+\tan^{-1}\left(\frac{x}{1+2\cdot 3\cdot x^2}\right)+$$
$$\ldots\ldots+\tan^{-1}\left(\frac{x}{1+(k+1)(k+2)x^2}\right)$$
$$= \tan^{-1}(k+2)x-\tan^{-1}x$$

whose L.H.S. $= \tan^{-1}(k+1)x-\tan^{-1}x+$
$$\tan^{-1}\left(\frac{x}{1+(k+1)(k+2)x^2}\right)$$

[using result (i)]

$$= \tan^{-1}(k+1)x-\tan^{-1}x+\tan^{-1}\left(\frac{(k+2)x-(k+1)x}{1+(k+1)(k+2)x^2}\right)$$
$$= \tan^{-1}(k+1)x-\tan^{-1}x+\tan^{-1}(k+2)x-\tan^{-1}(k+1)x$$
$$= \tan^{-1}(k+2)x-\tan^{-1}x = \text{R.H.S.}$$

Hence, the result is true for $n = k + 1$

Thus, by principle of mathematical induction, the result is true for all $n \in N$.

Q.2 If the vertices of the triangle ABC are represented by z_1, z_2, z_3 respectively, then show that the orthocentre of the triangle is

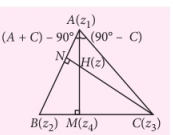
$$\frac{(a\sec A)z_1+(b\sec B)z_2+(c\sec C)z_3}{a\sec A+b\sec B+c\sec C}$$

– Md. Iqbal, Azamgarh

Ans. Let the foot of the perpendicular from A to BC be $M(z_4)$.

Then, we have $BM = AB\cos B = c\cos B$

and $CM = AC\cos C = b\cos C$



Therefore, we have

$$z_4=\frac{(b\cos C)z_2+(c\cos B)z_3}{b\cos C+c\cos B}$$

To find the complex representation of $H(z)$, we now need to know the ratio HA/HM.

We have

$$HA=\frac{AN}{\cos(A+C-90°)}=\frac{b\cos A}{\sin(A+C)}=\frac{b\cos A}{\sin B}$$

and $HM = CM\tan(90° - B) = b\cos C\cot B$

$$= \frac{b\cos C\cos B}{\sin B}$$

Therefore, we have

$$z = \frac{(HM)z_1 + (HA)z_4}{HM + HA}$$

$$= \frac{\left(\frac{b\cos C\cos B}{\sin B}\right)z_1 + \left[\left(\frac{b\cos A}{\sin B}\right)\frac{(b\cos C)z_2 + (c\cos B)z_3}{b\cos C + c\cos B}\right]}{\frac{b\cos C\cos B}{\sin B} + \frac{b\cos A\cos B}{\sin B}}$$

$$= \frac{\cos C\cos B(b\cos C + c\cos B)z_1 + b(\cos A\cos C)z_2 + (c\cos A\cos B)z_3}{(\cos A + \cos B\cos C)(b\cos C + c\cos B)}$$

$$= \frac{(a\cos B\cos C)z_1 + (b\cos A\cos C)z_2 + (c\cos A\cos B)z_3}{(b\cos C + c\cos B)\cos B\cos C + b\cos A\cos C + c\cos A\cos B}$$

$$= \frac{(a\sec A)z_1 + (b\sec B)z_2 + (c\sec C)z_3}{a\sec A + b\sec B + c\sec C}$$

[Putting $b\cos C + c\cos B = a$ and then dividing numerator and denominator by $\cos A\cos B\cos C$]

Q.3 Let $f(x) = \frac{1}{2} - \tan\left(\frac{\pi x}{2}\right)$, $-1 < x < 1$ and $g(x) = \sqrt{(3 + 4x - 4x^2)}$, then find the dom $(f + g)$.

– Suman Mehra, Raigarh

Ans. Given that $f(x) = \frac{1}{2} - \tan\left(\frac{\pi x}{2}\right)$, $-1 < x < 1$

Here domain of $f(x)$ is $d_1 = (-1, 1)$

and $g(x) = \sqrt{3 + 4x - 4x^2}$

$$= \sqrt{-(4x^2 - 6x + 2x - 3)}$$

$$= \sqrt{-(2x - 3)(2x + 1)}$$

$\Rightarrow$ $-(2x - 3)(2x + 1) \geq 0$

$\Rightarrow$ $(2x - 3)(2x + 1) \leq 0$

$\Rightarrow$ $-\frac{1}{2} \leq x \leq \frac{3}{2}$

$\therefore$ Domain of $g(x)$ is $d_2 = \left[-\frac{1}{2}, \frac{3}{2}\right]$

$\therefore$ Domain of $(f + g) = d_1 \cap d_2 = \left[-\frac{1}{2}, 1\right)$.

Q.4 Find the number of solutions of equation $\log_4(x - 1) = \log_2(x - 3)$.

– Harshit Gupta, A.P.

Ans. $\log_4(x - 1) = \log_2(x - 3)$

$\Rightarrow$ $\log_{2^2}(x - 1) = \log_2(x - 3)$

Using $\log_{b^n} a^m = \frac{m}{n}\log_b a$, we get

$$\frac{1}{2}\log_2(x - 1) = \log_2(x - 3)$$

$\Rightarrow$ $\log_2(x - 1)^{1/2} = \log_2(x - 3)$

$\Rightarrow$ $\sqrt{x - 1} = (x - 3)$ ..... (i)

On squaring (i) both sides, we get

$x - 1 = x^2 - 6x + 9 \Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow$ $x = 2$ or $x = 5 \Rightarrow x = 5$, neglecting $x = 2$.

Since, $\log(x - 3)$ is not defined for $x = 2$.

$\therefore$ Number of solutions is one.

Q.5 If $f : \{x, y, z\} \to \{a, b, c\}$ is a bijection. It is given that exactly one of the following statements (i), (ii), (iii) is true and the remaining two are false, *i.e.*,

(i) $f(x) = a$ (ii) $f(y) \neq a$

(iii) $f(z) \neq b$

Then, find $f^{-1}(a)$.

– Swastika, Bareilly

Ans. Case I : Assume statement (i) is true and (ii), (iii) are false.

$\Rightarrow$ $f(x) = a, f(y) = a, f(z) = b$

$\Rightarrow$ 'f' is not one-one function.

[$\because$ $f(x) = f(y) = a$]

$\therefore$ It is clear that (i) is false.

Case II : Assume that statement (ii) is correct and (i), (iii) are false.

$\Rightarrow$ $f(y) \neq a, f(x) \neq a, f(z) = b$.

i.e., where $f(x) \neq a, f(y) \neq a$

$\Rightarrow$ either $f(x) = b$ and $f(y) = c$

or $f(x) = c$ and $f(y) = c$

But in both the cases function is many-one.

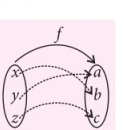
Case III : Assume that statement (iii) is correct and (i), (ii) are false.

$\Rightarrow$ $f(x) \neq a, f(y) = a, f(z) \neq b$

$f(x) \neq a, f(z) \neq b \Rightarrow f(x) = b, f(z) = c$

$\Rightarrow$ 'f' is one-one function.

$\therefore$ $f^{-1}(a) = y$



IMO Problems From TEAM SELECTION TESTS

* ALOK KUMAR, B.Tech, IIT Kanpur

1. A balance has arms of unequal length and pans of unequal weight. A certain article appears to weigh Q_1 or Q_2 according as it is put into one scale pan or the other. Similarly another article appears to weigh R_1 or R_2 in that order. Show that the true weight of the article which appears to weigh the same in whichever scale pan it is put is $\frac{Q_1R_2 - Q_2R_1}{Q_1 - Q_2 - R_1 + R_2}$.

2. Let $p(x) = x^2 + ax + b$ be a quadratic polynomial in which a and b are integers. Given any integer n, show that there is an integer m such that $p(n)p(n+1) = p(m)$.

3. If $f: R \to R$ is a function satisfying for all $x \in R$, $f(-x) = -f(x)$, $f(x+1) = f(x) + 1$ and $f\left(\frac{1}{x}\right) = \frac{f(x)}{x^2}$ (when $x \neq 0$) prove that $f(x) = x$ for all real values of x.

4. Suppose PQR is an equilateral triangle inscribed in a triangle ABC such that P lies on BC, Q on CA and R on AB. If $\frac{BP}{PC} = \frac{CQ}{QA} = \frac{AB}{RB}$. Prove that ΔABC is also equilateral.

5. Show that any circle with centre $(\sqrt{2}, \sqrt{3})$ cannot pass through more than one lattice point. (Lattice points are points with both coordinates integral)

6. A cyclic octagon $ABCDEFGH$ has sides a, a, a, b, b, b, b respectively. Find the radius of the circumcircle of this octagon.

7. Let AC and BD be two chords of a circle with centre O such that they intersect at right angles inside the circle at the point M. Suppose K and L are the mid-points of the chords AB and CD respectively. Prove that $OKML$ is a parallelogram.

8. M is an interior point of a triangle ABC. Bisectors of interior angles BMC, CMA, AMB intersect BC, CA, AB respectively at X, Y, Z. Show that AX, BY, CZ are concurrent; if P is the point of concurrence and $\frac{PA}{PX} \cdot \frac{PB}{PY} \cdot \frac{PC}{PZ} = 8$. Show that M is the circumcentre of ΔABC.

9. Define a sequence $(x_n)_{n=1}^{\infty}$ by

$x_1 = \sqrt{2}$; $x_{n+1} = \sqrt{2^{x_n}}$, $n \in N$.

Does this sequence converge? If so, find the limit.

10. An ant crawls at the rate of 10 cm per minute along a rubber band which can be stretched uniformly. Suppose that the rubber band is initially one metre long and it is stretched an additional metre at the end of each minute. If the ant begins at one end of the band, does it reach the other end? Why?

11. Show that $n^2 - 3n - 19$ is not a multiple of 289 for any integer n.

12. If $11 + 11\sqrt{11a^2 + 1}$ is an odd integer where a is a rational number, prove that a is a perfect square.

13. Find all natural numbers n for which the polynomial $x^n + 64$ can be written as a product of two non constant polynomials with integer coefficients.

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14. Let n, $n \geq 2$, be a positive integer. Let $x_1, x_2, ..., x_n$ be real numbers such that $\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n-1} x_i x_{i+1} = 1$.

Given an integer k, $1 \leq k \leq n$, determine the maximum value of $|x_k|$.

15. (a) For which positive integers n do there exist positive integers x, y, such that $\text{lcm}(x, y) = n!$, $\gcd(x, y) = 1998$?

(b) For which n is the number of such pairs x, y with $x \leq y$ less than 1998?

16. Let ABC be a triangle and P, Q points on the side AB such that the inradii of triangles APC and QBC are the same. Prove that the inradii of triangles AQC and PBC are also the same.

17. Let a, b, c be three real numbers and let

$$X = a + b + c + 2\sqrt{a^2 + b^2 + c^2 - ab - bc - ca}.$$

Show that $X \geq \max\{3a, 3b, 3c\}$, and that one of the numbers $\sqrt{X - 3a}$, $\sqrt{X - 3b}$, $\sqrt{X - 3c}$ is the sum of the other two.

18. Let P be a point in the interior of the convex quadrilateral $ABCD$. Show that at least one of the angles $\angle PAB, \angle PBC, \angle PCD, \angle PDA$ is less than or equal to $\pi/4$.

19. Let m, n be natural numbers with $m \geq n \geq 2$. Show that the number of polynomials of degree $2n - 1$ with distinct coefficients from the set $\{1, 2, ..., 2m\}$ which are divisible by $x^{n-1} + ... + x + 1$ is

$$2^n n! \left(4\binom{m+1}{n+1} - 3\binom{m}{n} \right).$$

20. Let $f_1, f_2, f_3 : R \to R$ be functions such that $a_1 f_1 + a_2 f_2 + a_3 f_3$ is monotonic for all $a_1, a_2, a_3 \in R$. Prove that there exist $c_1, c_2, c_3 \in R$, not all zero, such that $c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) = 0$ for all $x \in R$.

21. On each of 12 points around a circle we place a disk with one white side and one black side. We may perform the following move: select a black disk, and reverse its two neighbours. Find all initial configurations from which some sequence of such moves leads to the position where all disks but one are white.

22. For a natural number n, let $\phi(n)$ denote the number of natural numbers less than or equal to n and relatively prime to n, and let $\psi(n)$ be the number of prime factors of n. Show that if $\phi(n)$ divides $n - 1$ and $\psi(n) \leq 3$, then n is prime.

23. Find all integers (a, b, c, x, y, z) such that

$$a + b + c = xyz$$
$$x + y + z = abc$$

and $a \geq b \geq c \geq 1$, $x \geq y \geq z \geq 1$.

24. Let A_n denote the set of words of length n formed from the letters a, b, c which do not contain two consecutive letters both equal to a or both equal to b. Let B_n denote the set of words of length n formed from the letters a, b, c which do not contain three consecutive letters which are pairwise distinct. Prove that $|B_{n+1}| = 2|A_n|$ for all $n \geq 1$.

25. Let $m \geq 2$ be an integer. Find the smallest integer $n > m$ such that for any partition of the set $\{m, m + 1, ..., n\}$ into two subsets, at least one subset contains three numbers a, b, c (not necessarily different) such that $a^b = c$.

26. Find all positive integers (x, n) such that $x^n + 2^n + 1$ is a divisor of $x^{n+1} + 2^{n+1} + 1$.

27. The side lengths of a triangle and the diameter of its incircle are four consecutive integers. Find all such triangles.

SOLUTIONS

1. Let the two pans be of weights P_1, P_2 and lengths l_1, l_2 respectively. Let X be the actual weight of the third object and S be its apparent weight. Then $(X + P_1)l_1 + (S + P_2)l_2$ and $(S + P_1)l_1 = (X + P_2)l_2$. Subtracting, we get $(X - S)l_1 = (S - X)l_2$; therefore $X = S$.

So we have $(X + P_1)l_1 = (X + P_2)l_2$ ... (1)

Now let y be the actual weight of the first object. Then

$$(Y + P_1)l_1 = (Q_1 + P_2)l_2 \quad \text{... (2)}$$
$$(Q_2 + P_1)l_1 = (Y + P_2)l_2 \quad \text{... (3)}$$

Now (1) – (2) and (1) – (3) implies

$(X - Y)l_1 = (X + Q_1)l_2$

and $(X - Q_2)l_1 = (X - Y)l_2$

From these two relations, we get

$$\frac{X-Q_1}{X-Q_2}=\frac{l_1^2}{l_2^2}$$

Similarly, we have $\frac{X-R_1}{X-R_2}=\frac{l_1^2}{l_2^2}$.

Therefore $\frac{X-Q_1}{X-Q_2}=\frac{X-R_1}{X-R_2}$; *i.e*;

$$X^2 - X(Q_1 + R_2) + Q_1R_2 = X^2 - X(Q_2 + R_1) + Q_2R_1$$

and hence $X=\frac{Q_1R_2 - Q_2R_1}{Q_1 - Q_2 - R_1 + R_2}$.

2. Since $p(n)p(n + 1)$ is of fourth degree n with leading coefficient 1, m, if it exists, must be of the form $n^2 + tn + s$

Therefore $p(0)\ p(1) = p(s)$

$\Rightarrow\ b^2 + ab + b = s^2 + as + b$

$\Rightarrow\ s^2 - b^2 = a(b - s)$

$\Rightarrow\ (s - b)(s + b) = a(b - s)$

$\Rightarrow\ (s - b)(s + a + b) = 0$

Thus we have $s = b$ or $s = -(a + b)$.

Taking $s = b$, we have

$(n^2 + an + b)((n + 1)^2 + a(n + 1) + b)$

$= (n^2 + tn + b)^2 + a(n^2 + tn + b) + b$

Equating the coefficients of n^3, we obtain $t = a + 1$.

Taking $m = n^2 + (a + 1)n + b$, it can be routinely verified that $p(n)p(n + 1) = p(m)$ for all $n \in N$.

Alternative solution : Writing $p(x) = (x + \alpha)(x + \beta)$, we have $\alpha + \beta = a$ and $\alpha\beta = b$. Now

$$(n+\alpha)(n+1+\beta) = n^2 + (\alpha+\beta+1)n + \alpha\beta + \alpha$$

$$= n^2 + (a + 1)n + b + \alpha$$

Similarly $(n + \beta)(n + 1 + \alpha) = m + \beta$

where $m = n^2 + (a + 1)n + b$. Therefore

$$p(n)p(n+1) = (n+\alpha)(n+\beta)(n+1+\alpha)(n+1+\beta)$$

$$= (m + \alpha)(m + \beta) = p(m)$$

3. Put $f(x) = x + g(x)$; then g has the properties :

$g(-x) = -g(x)$,

$g(x+1) = g(x)$ and when $x \neq 0, g\left(\frac{1}{x}\right)=\frac{g(x)}{x^2}$.

We have to show that $g(x) = 0$ for all $x \in R$.

$g(0) = -g(0) \ \Rightarrow\ g(0) = 0$.

$g(-1) = g(-1 + 1) = g(0) = 0$.

When x is neither 0 nor –1,

$$g(x) = x^2 g\left(\frac{1}{x}\right) = x^2 g\left(\frac{x+1}{x}\right)$$

$$= x^2\,\frac{(x+1)^2}{x^2}\,g\left(\frac{x}{x+1}\right) = (x+1)^2 g\left(\frac{x}{x+1} - 1\right)$$

$$= (x+1)^2 g\left(\frac{-1}{x+1}\right) = -(x+1)^2 g\left(\frac{1}{x+1}\right)$$

$$= -g(x + 1) = -g(x)$$

Therefore $g(x) = 0$.

4. Let z_1, z_2, z_3 be the affixes of A, B, C. Let $\frac{BP}{PC}=\frac{CQ}{QA}=\frac{AR}{RB}=\lambda$

Then $\frac{z_2+\lambda z_3}{1+\lambda}, \frac{z_3+\lambda z_1}{1+\lambda}$ and $\frac{z_1+\lambda z_2}{1+\lambda}$ are respectively the affixes of P, Q and R. We can assume that the circumcentre of ΔPQR is the origin.

Then $\omega(z_2 + \lambda z_3) = (z_3 + \lambda z_1)$ where $\omega = e^{2i\pi/3}$, a cube root of unity. So we have

$\omega z_2 - z_3 = \lambda(z_1 - \omega z_3)$

Similarly, $\omega z_3 - z_1 = \lambda(z_2 - \omega z_1)$

and $\omega z_1 - z_2 = \lambda(z_3 - \omega z_2)$

Therefore $\omega z_2 - z_3 = \lambda(z_1 - \omega z_3)$

$= \lambda^2(\omega z_1 - z_2) = \lambda^3(z_3 - \omega z_2)$

$\Rightarrow\ (1 + \lambda^3)(\omega z_2 - z_3) = 0$

$\Rightarrow\ \omega z_2 = z_3$.

Similarly $\omega z_3 = z_1$

Therefore the triangle ABC is equilateral.

5. Suppose (a, b) and (c, d) are two lattice points on a circle with $(\sqrt{2}, \sqrt{3})$ as centre. Then $(a-\sqrt{2})^2 + (b-\sqrt{3})^2 = (c-\sqrt{2})^2 + (d-\sqrt{3})^2$

Simplifying we get

$$a^2 + b^2 - c^2 - d^2 - 2(\sqrt{2}(a-c) + \sqrt{3}(b-d)) = 0\,.$$

Therefore it follows that $a = c$ and $b = d$. Hence any circle with $(\sqrt{2}, \sqrt{3})$ as centre can pass through at most one lattice point.

Alternative solution : Let P and Q be two lattice points and R be the mid-point of PQ. Let O be the centre of the circle. Since the slope of OR is irrational whereas that of PQ is not, OR is not perpendicular to PQ. Hence both P and Q can't be on the circle.

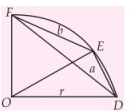
6. Let r be the radius of the circumcircle. Since 4 sides (including DE) are of length a and other sides-EF is one of them-are of length b,

$4\angle DOE + 4\angle EOF = 2\pi$ *i.e.* $\angle DOF = \dfrac{\pi}{2}$

Now, $2\angle DEF = \angle DEF + \angle OED + \angle OEF$

$= \angle DEF + \angle ODE + \angle OFE = 2\pi - \dfrac{\pi}{2} = \dfrac{3\pi}{2}$

$= \angle DEF = \dfrac{3\pi}{4}$



Therefore $2r^2 = FD^2 = a^2 + b^2 - 2ab\cos\dfrac{3\pi}{4}$

$= a^2 + b^2 - 2ab\left(-\dfrac{1}{\sqrt{2}}\right) = a^2 + b^2 + \sqrt{2}ab$

$\Rightarrow r = \sqrt{\dfrac{a^2 + b^2 + \sqrt{2}ab}{2}}$

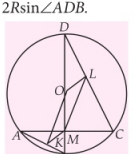
7. **1st solution :** Choose M as the origin. Let A, B, C, D be $(2a, 0)$, $(0, 2b)$, $(2c, 0)$, $(0, 2d)$ respectively. Then

$AM \cdot MC = BM \cdot MD \Rightarrow ac = bd$

(in magnitude as well as in sign).

O being the intersection of the perpendicular bisectors of AC and BD is $(a + c, b + d)$. Since K, L are (a, b), (c, d) respectively, the midpoint of KL is $\left(\dfrac{a+c}{2}, \dfrac{b+d}{2}\right)$ which is also the mid-point of OM. Therefore $OKML$ is a parallelogram.

2nd solution : Let R be the radius of the circle. Then in ΔADB, $AB = 2R\sin\angle ADB$.



Also in ΔCAD, $CD = 2R\sin\angle CAD = 2R\cos\angle ADB$

$(\because$ In $\Delta ADM, \angle ADB + \angle CAD = 90°)$

Therefore $AB^2 + CD^2 = 4R^2$.

Now, $AK^2 + OK^2 = AO^2 \Rightarrow \dfrac{1}{2}AB^2 + OK^2 = R^2$

and therefore $OK = \dfrac{1}{2}CD = ML$ ($\because$ in the right angled triangle CMD, L is the mid-point of the hypotenuse). Similarly $OL = KM$.

3rd solution : Let $\angle KAM = \theta$.

In ΔAKM, $AK = KM \Rightarrow \angle AMK = \theta$

$\therefore \angle AMK = 180° - 2\theta$.

So, $\angle OKM = 180° - 2\theta - 90° = 90° - 2\theta$.

By similar argument, since $\angle MDC = \theta$,

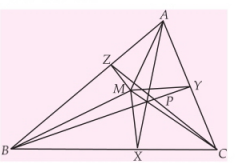
we get $\angle OLM = 90° - 2\theta$. Further,

$\angle KML = \angle KMA + \angle AMD + \angle DML$

$= \theta + 90° + \theta = 90° + 2\theta$

Thus, three angles $\angle OKM$, $\angle OLM$, $\angle KML$ of $OKML$ are $90° - 2\theta$, $90° - 2\theta$, $90° + 2\theta$ respectively; so the remaining is $90° + 2\theta$ and hence $OKML$ is a parallelogram.

8. Since MX is the internal bisector of $\angle BMC$, $\dfrac{BX}{XC} = \dfrac{MB}{MC}$; similarly $\dfrac{CY}{YA} = \dfrac{MC}{MA}$ and $\dfrac{AZ}{ZB} = \dfrac{MA}{MB}$ therefore $\dfrac{BX}{XC} \cdot \dfrac{CY}{YA} \cdot \dfrac{AZ}{ZB} = 1$.



Hence by Ceva theorem, AX, BY, CZ are concurrent.

Let $\Delta BPC = t_1$, $\Delta APC = t_2$ and $\Delta APB = t_3$

Then,

$\dfrac{PA}{PX} = \dfrac{\Delta APB}{\Delta BPX} = \dfrac{\Delta APC}{\Delta PXC} = \dfrac{\Delta APB + \Delta APC}{\Delta BPX + \Delta PXC} = \dfrac{t_2 + t_3}{t_1}$

Similarly $\dfrac{PB}{PY} = \dfrac{t_3 + t_1}{t_2}$ and $\dfrac{PC}{PZ} = \dfrac{t_1 + t_2}{t_3}$.

Now,

$8 = \dfrac{(t_2 + t_3)(t_3 + t_1)(t_1 + t_2)}{t_1 t_2 t_3} \geq \dfrac{2\sqrt{t_2 t_3}.2\sqrt{t_3 t_1}.2\sqrt{t_1 t_2}}{t_1 t_2 t_3}$

(Equality holds only when $t_1 = t_2 = t_3 = 8$)

Thus $t_1 = t_2 = t_3$. Now $\frac{BX}{XC} = \frac{\Delta ABX}{\Delta ACX}$

$= \frac{\Delta PBX}{\Delta PCX} = \frac{\Delta ABX - \Delta PBX}{\Delta ACX - \Delta PCX} = \frac{t_3}{t_2} = 1$

i.e., $BX = XC$. Since MB is the bisector of $\angle BMC$ it follows that $MB = MC$. Similarly $MC = MA$. Therefore M is the circumcentre. (Note that P is the centroid).

9. Since $x_2 = \sqrt{2}^{\sqrt{2}} > \sqrt{2} = x_1$ and for any integer $n > 1$,

$$x_n > x_{n-1} \Rightarrow \sqrt{2}^{x_n} > \sqrt{2}^{x_{n-1}} \Rightarrow x_{n+1} > x_n,$$

it follows by induction that $(x_n)_{n=1}^{\infty}$ is an increasing sequence. Also by induction it is easy to see that $(x_n)_{n=1}^{\infty}$ is bounded above by 2. Therefore this sequence has a limit, say l. Now the defining relation of the sequence gives

$$l = \sqrt{2}^{l} = (2^{1/2})^{l} = (1+1)^{l/2} \geq 1 + \frac{l}{2}$$

Therefore $\frac{l}{2} \geq 1 \Rightarrow l \geq 2$. Hence $l = 2$.

10. Suppose the ant never reaches the other end. Let the length of the rubber band after n^{th} stretching be b_n cm; then $b_n = 100 + 10n = 10(n + 10)$. Let the ant be at the distance a_n cm, from the starting point, just after the stretching at the end of n minutes. After one more minute it is at the distance $a_n + 10$, then by stretching, it is at the distance of a_{n+1}. Since the stretching is uniform, we have

$$\frac{a_{n+1}}{a_n + 10} = \frac{b_{n+1}}{b_n} = \frac{n+2}{n+1} \Rightarrow \frac{a_{n+1}}{n+2} = \frac{a_n}{n+1} + \frac{10}{n+1}$$

$$\therefore \quad \sum_{k=0}^{n} \frac{a_{k+1}}{k+2} = \sum_{k=0}^{n} \left(\frac{a_k}{k+1} + \frac{10}{k+1} \right)$$

$$\Rightarrow \quad \frac{a_{n+1}}{n+2} = 10 \sum_{k=0}^{n} \frac{1}{k+1} = 10 \sum_{k=1}^{n+1} \frac{1}{k}$$

$$\Rightarrow \quad a_{n+1} = 10(n+2) \sum_{k=1}^{n+1} \frac{1}{k}$$

Since $a_{n+1} < b_{n+1}$,

$$10(n+2) \sum_{k=1}^{n+1} \frac{1}{k} < 100(n+2) \Rightarrow \sum_{k=1}^{n+1} \frac{1}{k} < 10.$$

i.e., $1 + \frac{1}{2} + \frac{1}{3} + \ldots$ is convergent, a contradiction. Therefore the ant reaches the other end of the band.

11. **1st solution :** Suppose $17^2 | n^2 - 3n - 19$.

Since $n^2 - 3n - 19 = (n + 7)(n - 10) + 51$,

$17 | (n + 7)(n - 10)$; $\therefore$ $17^2 | (n + 7)(n - 10)$

($\because$ $x + 7 \equiv n - 10 \pmod{17}$)

$\Rightarrow$ $17^2 | (n^2 - 3n - 19) - (n + 7)(n - 10)$

i.e., $17^2 | 51$, a contradiction.

Consequently $n^2 - 3n - 19$ is not a multiple of 289.

2nd solution : Suppose for some $k \in Z$,

$n^2 - 3n - 19 = 289k$. Then $n^2 - 3n - (19 + 298k) = 0$;

therefore $n = \frac{1}{2}(3 \pm \sqrt{9 + 4(19 + 289k)})$

$= \frac{1}{2}(3 \pm \sqrt{85 + 4 \cdot 289k}) = \frac{1}{2}(3 \pm \sqrt{17(5 + 69k)})$

But $5 + 68k$ is not a multiple of 17 and hence $17(5 + 68k)$ cannot be a perfect square. Thus $289 | n^2 - 3n - 19$.

12. Let $\lambda = 11 + 11\sqrt{11a^2 + 1}$.

Then $(\lambda - 11)^2 = 11^2(11a^2 + 1)$;

Simplifying we get $\lambda(\lambda - 22) = 11^3 a^2$.

Putting $|a| = \frac{r}{s}$ where $r, s \in N$ such that $(r, s) = 1$,

gives $\lambda(\lambda - 22)s^2 = 11^3 r^2$. Since $11^2 \nmid s$ because otherwise 11 would divide r, $11 \mid \lambda$. Writing $\lambda = 11\mu$, we get $\mu(\mu - 2)s^2 = 11r^2$. Since $11 \nmid s$ for otherwise we would have $11 \mid r$, it follows that $s = 1$. Thus we have $\mu(\mu - 2) = 11r^2$.

Since $\mu - 2$ and μ are consecutive odd integers, they are relatively prime. If $11 \mid \mu - 2$, then μ is a square of the form $11n + 2$ which is not possible. Therefore $11 | \mu$ and hence $\mu = 11n^2$ for some $n \in N$.

Thus, we have $\lambda = 11\mu = 11^2 n^2$.

13. We claim that $n = 3k$ or $n = 4k$ for some positive integer k. Suppose that $x^n + 64 = f(x)\, g(x)$ for two non constant integer polynomials $f(x)$ and $g(x)$. Let

$a = \sqrt[n]{64} = \alpha_k = \frac{(2k+1)\pi}{n}$ for $k = 1, 2, \ldots, n$. Then

$f(x)\,g(x) = x^n + 64$

$= (x - a\,cis\alpha_1)(x - a\,cis\alpha_2)...(x - a\,cis\alpha_n)$

Since there is at most one real root for $x^n + 64$ (when n is odd), one of $f(x)$ or $g(x)$ has only complex roots. WLOG, let $f(x)$ be that polynomial. Let c be the constant term of $f(x)$ and d be the degree of $f(x)$. Since $g(x)$ is non constant, $d < n$. Since $f(x)$ has integer coefficients, all its roots can be paired as conjugates. So $2m = d < n$.

Noticing $(a\,cis\alpha_k)\overline{(a\,cis\alpha_k)} = 64^{2/n}$.

We have $c = 64^{\frac{2m}{n}} = (2^6)^{\frac{2m}{n}}$ an integer less than 64. Thus $\frac{2m}{n}$ can be reduced to a fraction with its denominator equal to 2, 3, or 6, so $3|n$ or $4|n$.

Since $x^{3k} + 64 = (x^k + 4)(x^{2k} - 4x^k + 16)$

$x^{4k} + 64 = (x^{2k} + 4x^k + 8)(x^{2k} - 4x^k + 8)$,

both $n = 3k$ and $n = 4k$ satisfy the condition of the problem. Therefore n is a multiple of 3 or 4, as claimed.

14. **1st solution:** The answer is

$$|x_k|_{\max} = \sqrt{\frac{2k(n+1-k)}{n+1}}.$$

We have

$x_1^2 + (x_1 + x_2)^2 + (x_2 + x_3)^2 + ... + (x_{n-1} + x_n)^2 + x_n^2 = 2$

From the RMS-AM inequality, we have

$$\sqrt{\frac{x_1^2 + (x_1 + x_2)^2 + ... + (x_{k-1} + x_k)^2}{k}}$$

$$\geq \frac{|x_1| + |x_1 + x_2| + ... + |x_{k-1} + x_k|}{k}$$

$$\geq \frac{|x_1 - (x_1 + x_2) + ... + (-1)^{k-1}(x_{k-1} + x_k)|}{k}$$

$$= \frac{|x_k|}{k}$$

$$\Rightarrow x_1^2 + (x_1 + x_2)^2 + ... + (x_{k-1} + x_k)^2 \geq \frac{x_k^2}{k}.$$

Similarly, we have

$$(x_k + x_{k+1})^2 + ... + (x_{n-1} + x_n)^2 + x_n^2 \geq \frac{x_k^2}{n-k+1}.$$

Adding the above two inequalities, we have

$$2 = x_1^2 + (x_1 + x_2)^2 + ... + (x_{n-1} + x_n)^2 + x_n^2$$

$$\geq \left(\frac{1}{k} + \frac{1}{n-k+1}\right)x_k^2,$$

i.e., $|x_k| \leq |x_k|_{\max}$. Equality holds if and only if

$x_1 = -(x_1 + x_2) = x_2 + x_3 = ... = (-1)^{k-1}(x_{k-1} + x_k)$

and $x_k + x_{k+1} - (x_{k+1} + x_{k+2}) = ... = (-1)^{n-1}x_n$.

Thus $|x_k|$ obtains its maximum value $|x_k|_{\max}$ if and only if

$$x_i = \begin{cases} (-1)^{k-i}\dfrac{x_k i}{k} & , \text{if } i = 1, 2, ..., k-1 \\ (-1)^{i-k}\dfrac{x_k(n+1-i)}{n-k+1} & , \text{if } i = k+1, ..., n \end{cases}$$

2nd solution: We consider the quadratic form

$$\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} x_i x_{i+1}$$

For a fixed positive k, $1 \leq k \leq n$, we complete squares from both ends towards x_k. We do this by finding real numbers $a_1, a_2, ..., a_n$ such that

$$1 = \sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n-1} x_i x_{i+1}$$

$$= \left(\sqrt{a_1}x_1 + \sqrt{1-a_2}x_2\right)^2 + \left(\sqrt{a_2}x_2 + \sqrt{1-a_3}x_3\right)^2$$

$$+ ... + \left(\sqrt{a_{k-1}}x_{k-1} + \sqrt{1-a_k}x_k\right)^2$$

$$+ \left(\sqrt{1-a_{n+1-k}}x_k + \sqrt{a_{n-k}}x_{k+1}\right)^2 + ...$$

$$..... + \left(\sqrt{1-a_2}x_{n-1} + \sqrt{a_1}x_n\right)^2$$

$$..... + [1 - (1-a_k) - (1-a_{n+1-k})]x_k^2$$

Here $a_1 = 1$, $2\sqrt{a_i(1-a_{i+1})} = 1$,

For $i = 1, 2, ..., \max(k-1, n-k)$.

From induction, we can prove that

$$a_i = \frac{i+1}{2i} \text{ and } 1 - a_i = \frac{i-1}{2i}$$

For all the i. Thus $[1 - (1-a_k) - (1-a_{n+1-k})]x_k^2 \leq 1$

and $|x_k| = \sqrt{\dfrac{1}{1 - (1-a_k) - (1-a_{n+1-k})}} = |x_k|_{\max}$

if and only if $\sqrt{a_1}x_1 + \sqrt{1-a_2}x_2$

$= \sqrt{a_2}x_2 + \sqrt{1-a_3}x_3 = \ldots = \sqrt{a_{k-1}}x_{k-1} + \sqrt{1-a_k}x_k$

$= \sqrt{1-a_{n+1-k}}x_k + \sqrt{a_{n-k}}x_{k+1} = \ldots$

$= \sqrt{1-a_2}x_{n-1} + \sqrt{a_1}x_n = 0$

Solving the above equations for $x_1, x_2, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n$ yields the values of the x_i's which produce the desired maximum value $|x_k|_{\max}$ for $|x_k|$.

15. (a) Let $x = 1998a$, $y = 1998b$. So a, b, are positive integers such that $a < b$, $\gcd(a, b) = 1$.

We have $\text{lcm}(x, y) = 1998ab = 2 \cdot 3^3 \cdot 37ab = n!$. Thus $n \geq 37$ and it is easy to see that this condition is also sufficient.

(b) The answers are $n = 37, 38, 39, 40$. We only need to consider positive integers $n \geq 37$. For $37 \leq n < 41$, let $k = ab = \dfrac{n!}{1998}$. Since $\gcd(a, b) = 1$, any prime factor of k that occurs in a cannot occur in b, and vice-versa. There are 11 prime factors of k, namely 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31. For each of those prime factors, one must decide only whether it occurs in a or in b. These 11 decisions can be made in a total of $2^{11} = 2048$ ways. However, only half of these ways will satisfy the condition $a < b$. Thus there will be a total of 1024 such pairs of (x, y) for $n = 37, 38, 39, 40$. Since 41 is a prime, we can see by a similar argument that there will be at least 2048 such pairs of (x, y) for $n \geq 41$.

16. **1st solution:** We first establish following lemma.

Lemma : Let XYZ be a triangle, let r be the inradius of XYZ, let h_x be the length of the altitude corresponding to base YZ. Then

$$\frac{h_x - 2r}{h_x} = \tan\frac{\angle Y}{2}\tan\frac{\angle Z}{2}.$$

Proof : Let w and I be the incircle and the incenter of XYZ, respectively. Let w and YZ meet at S. Let Y_1 and Z_1 be points on XY and XZ, respectively, such that Y_1Z_1 is tangent to w at T and $Y_1Z_1 \| YZ$. We have

$$\angle TY_1I = \frac{180^\circ - \angle XYZ}{2},\ \angle TZ_1I = \frac{180^\circ - \angle XZY}{2}$$

$$\Rightarrow \frac{h_x - 2r}{h_x} = \frac{Y_1Z_1}{YZ} = \frac{\cot\angle TY_1I + \cot\angle TZ_1I}{\cot\dfrac{\angle XYZ}{2} + \cot\dfrac{\angle XZY}{2}}$$

$$= \frac{\tan\dfrac{\angle XYZ}{2} + \tan\dfrac{\angle XZY}{2}}{\cot\dfrac{\angle XYZ}{2} + \cot\tan\dfrac{\angle XZY}{2}}$$

$$= \tan\frac{\angle XYZ}{2} + \tan\frac{\angle XZY}{2}, \text{ as desired.}$$

Now we prove our main result. Let h_c be the length of the altitude corresponding to side AB. Let r_1, r_2, r_3, r_4 be the inradii of triangles CAP, CBQ, CAQ, CBP, respectively. Since $r_1 = r_2$, from the lemma, we have $\dfrac{h_c - 2r_1}{h_c} = \dfrac{h_c - 2r_2}{h_c}$

$$\tan\frac{\angle CAP}{2}\tan\frac{\angle APC}{2} = \tan\frac{\angle CBQ}{2}\tan\frac{\angle BQC}{2}$$

$$\tan\frac{\angle CAP}{2}\cot\frac{\angle BPC}{2} = \tan\frac{\angle CBQ}{2}\cot\frac{\angle AQC}{2}$$

$$\tan\frac{\angle CAQ}{2}\tan\frac{\angle AQC}{2} = \tan\frac{\angle CBP}{2}\tan\frac{\angle BPC}{2}$$

$$\frac{h_c - 2r_3}{h_c} = \frac{h_c - 2r_4}{h_c}$$

$r_3 = r_4$, as desired.

2nd solution: We provide an alternate proof to the lemma. Define w, I, Y_1, Z_1 as before. Triangles XY_1Z_1 and XYZ are similar, so the ratio $\dfrac{(h_x - 2r)}{(h_x)}$ is the ratio between the corresponding altitudes of the triangles, and hence it is equal to the ratio between the inradii of the triangles.

Let r_1, K_1 and s_1 be the inradius, area and semiperimeter of triangle XY_1Z_1, respectively. Also let $x_1 = Y_1Z_1$, $y_1 = Z_1X$, and $z_1 = Y_1X$. Since w is an excircle of triangle XY_1Z_1, we have

$$K_1 = r(s_1 - x_1) = r_1s_1 = \sqrt{s_1(s_1 - x_1)(s_1 - y_1)(s_1 - z_1)}$$

Together, these equations imply that

$$\frac{r_1}{r} = \frac{s_1 - x_1}{s_1} = \frac{r_1}{s_1 - y_1} \cdot \frac{r_1}{s_1 - z_1} = \tan\frac{\angle Y}{2}\tan\frac{\angle Z}{2},$$

as desired.

17. We have $3(b-c)^2 \geq 0$, which implies that $4b^2 + 4c^2 - 4bc \geq b^2 + c^2 + 2bc$. Thus

$$4(a^2+b^2+c^2-ab-bc-ca) \geq (2a-b-c)^2$$

$$\Rightarrow 2\sqrt{a^2+b^2+c^2-ab-bc-ca} \geq |2a-b-c| \geq 2a-b-c$$

$$\Rightarrow X \geq 3a$$

Similarly, $X \geq 3b$ and $X \geq 3c$, so $X \geq \max\{3a, 3b, 3c\}$.

Now, let $u=\sqrt{X-3a}$, $v=\sqrt{X-3b}$, $w=\sqrt{X-3c}$.

Then $a=\dfrac{(X-u^2)}{3}$, $b=\dfrac{(X-v^2)}{3}$ and $c=\dfrac{(X-w^2)}{3}$

We have

$$X = a+b+c+2\sqrt{a^2+b^2+c^2-ab-bc-ca}$$

$$\Leftrightarrow X = a+b+c+2\sqrt{\frac{1}{2}(a-b)^2+\frac{1}{2}(b-c)^2+\frac{1}{2}(c-a)^2}$$

$$\Leftrightarrow u^2+v^2+w^2 = 2\sqrt{\frac{1}{2}(u^2-v^2)^2+\frac{1}{2}(v^2-w^2)^2+\frac{1}{2}(w^2-u^2)^2}$$

$$\Leftrightarrow u^2+v^2+w^2 = 2\sqrt{u^4+v^4+w^4-u^2v^2-v^2w^2-w^2u^2}$$

$$\Leftrightarrow u^4+v^4+w^4-2u^2v^2-2v^2w^2-2w^2u^2=0$$

$$\Leftrightarrow u^4+2u^2v^2+v^4-2w^2(u^2+v^2)+w^4-4u^2v^2=0$$

$$\Leftrightarrow (u^2+v^2)^2-2w^2(u^2+v^2)+w^4-4u^2v^2=0$$

$$\Leftrightarrow (u^2+v^2-w^2)^2-(2uv)^2=0$$

$$\Leftrightarrow (u^2+v^2+2uv-w^2)(u^2+v^2-2uv-w^2)=0$$

$$\Leftrightarrow ((u+v)^2-w^2)((u-v)^2-w^2)=0$$

$$\Leftrightarrow (u+v+w)(u+v-w)(u-v+w)(u-v-w)=0$$

From definition, $u, v, w \geq 0$.

If $uvw = 0$, then $X = 3a = 3b = 3c$ and we are done. If $uvw \neq 0$, then $u + v + w > 0$ and one is the sum of the other two, as desired.

18. **1st solution:** This is a generalization of a special property of the Brocard points of a triangle.

Let $a = AB$, $b = BC$, $c = CD$, $d = DA$;
$w = AP$, $x = BP$, $y = CP$, $z = DP$;
$\alpha = \angle PAB$, $\beta = \angle PBC$, $\gamma = \angle PCD$, $\delta = \angle PDA$.

Let XYZ be a triangle and $XY = z$, $YZ = x$, $ZX = y$. By "smoothing" pairs of adjacent side lengths of XYZ to their average, we keep the perimeter constant while not decreasing the triangle's area. Actually we can fix X and Y and find a point Z' such that Z' is one the perpendicular bisector of XY and $2Z'X = x + y$. Applying Heron's formula, we have

$$16[XYZ]^2 = (y+z-x)(z+x-y)(x+y-z)(x+y+z)$$

$$16[XYZ']^2 = z^2(x+y-z)(x+y+z)$$

Note that

$$(y+z-x)(z+x-y) = z^2-(x-y)^2 \leq z^2$$

gives $[XYZ] \leq [XYZ']$.

So by smoothing AB and BC; CD and DA; BC and CD; and DA and AB in that order, we form a rhombus then a square with the same perimeter and at least as much as area as the original quadrilateral. The area of this square is

$$\left(\frac{a+b+c+d}{4}\right)^2 \leq \frac{a^2+b^2+c^2+d^2}{4}$$

by the root-mean-square inequality. Thus

$$a^2+b^2+c^2+d^2 \geq 4[ABCD]$$
$$= 4([PAB]+[PBC]+[PCD]+[PDA])$$
$$= 2(aw\sin\alpha + bx\sin\beta + cy\sin\gamma + dz\sin\delta)$$

Also, applying the law of cosines on triangles PAB, PBC, PCD, PDA on angles $\alpha, \beta, \gamma, \delta$, we have

$$x^2 = a^2 + w^2 - 2aw\cos\alpha$$

and three other similar expressions. Summing these gives

$$2(aw\cos\alpha + bx\cos\beta + cy\cos\gamma + dz\cos\delta) = a^2+b^2+c^2+d^2 \geq 2(aw\sin\alpha + bx\sin\beta + cy\sin\gamma + dz\sin\delta).$$

Now when $45° < \theta < 180°$, $\sin\theta > \cos\theta$. So if $\alpha, \beta, \gamma, \delta$ were all greater than 45° then the above inequality would be false. Therefore one must be less than or equal to 45°, as desired.

2nd solution: We are going to use a more algebraic approach to prove

$$4[ABCD] \leq a^2+b^2+c^2+d^2$$

and the rest is the same. Under the same notations, we have

$$2[ABCD] = 2[ABC] + 2[ACD] = ab\sin B + cd\sin D$$

and $4[ABCD]^2 = a^2b^2\sin^2B + c^2d^2\sin^2D + 2abcd\sin B\sin D$

Applying law of cosines to triangles ABC and ACD, we have $a^2 + b^2 - 2ab\cos B = AC^2 = c^2 + d^2 - 2cd\cos D$, which implies that

$$\frac{(a^2+b^2-c^2-d^2)^2}{4} = a^2b^2\cos^2 B + c^2d^2\cos^2 D - 2abcd\cos B\cos D$$

Thus, $4[ABCD]^2 + \frac{(a^2+b^2-c^2-d^2)^2}{4} = a^2b^2 + c^2d^2 - 2abcd\cos(B+D)$

$\Rightarrow 4[ABCD]^2 \le a^2b^2 + c^2d^2 + 2abcd = (ab+cd)^2$

$\Rightarrow 2[ABCD] \le ab + cd \le \frac{a^2+b^2}{2} + \frac{c^2+d^2}{2}$

$\Rightarrow 4[ABCD] \le a^2 + b^2 + c^2 + d^2$

19. For $n \ge 2$, let $h(x) = x^{n-1} + ... + x + 1$ and let $f(x) = a_{2n-1}x^{2n-1} + ... + a_1x + a_0$ be a degree $(2n-1)$ polynomial. First we claim that $f(x)$ is divisible by $h(x)$ if and only if

$$a_{2n-1} + a_{n-1} = a_{2n-2} + a_{n-2} = ... = a_n + a_0$$

For $1 \le i \le n-1$, let w_i be distinct the nth roots of unity other than 1. Since $h(x)(x-1) = x^n - 1$, the w_i are the roots to $h(x)$. So, $h(x)|f(x)$ if and only if $f(w_i) = 0$ for all w_i. Let

$$g(x) = (a_{2n-1} + a_{n-1})x^{n-1} + (a_{2n-2} + a_{n-2})x^{n-2} + ... + (a_n + a_0)$$

Since $w_i^n = 1$, we have

$$w_i^{2n-1} = w_i^{n-1},\ w_i^{2n-2} = w_i^{n-2},\ ...,\ \text{and}\ w_i^n = w_i^0$$

Then $g(w_i) = f(w_i)$ for all i, so $h(x)$ divides $f(x)$ if and only if it divides $g(x)$. But this occurs if and only if

$$a_{2n-1} + a_{n-1} = a_{2n-2} + a_{n-2} = ... = a_n + a_0$$

as claimed.

Suppose that there are N sets of n pairs of distinct integers from 1, 2, ..., $2m$ such that the n pairs have the same sum. In any set, the pairs are disjoint since the numbers in each pair have the same sum. Then there are N sets to choose from and $n!$ ways to choose which pairs from the set correspond to which pairs (a_i, a_{i-n}). Also, for each of the n pairs, there are 2 ways to assign the values. Therefore, there are a total of $2^n\, n!N$ satisfactory polynomials. Now we only need to prove that $N = 4\binom{m+1}{n+1} - 3\binom{m}{n}$.

Let $S = \{1, 2, ..., 2m\}$. For a positive integer $k \le 2m$, there are $\left\lfloor \frac{(k-1)}{2} \right\rfloor$ pairs of distinct integers from S that add up to k: the $k-1$ pairs $\{1, k-1\}, \{2, k-2\}$ ..., $\{k-1, 1\}$ count each pair twice as well as a possible $\left\{\frac{k}{2}, \frac{k}{2}\right\}$ pair. If $4m > k > 2m$, then there are $\left\lfloor \frac{(4m-k+1)}{2} \right\rfloor$ pairs of distinct integers from $[2m]$ that add up to k by a similar argument. As k ranges from $3 = 1 + 2$ to $4m - 1 = (2m-1) + 2m$, the numbers of pairs of distinct integers from $[2m]$ adding up to k are $1, 1, 2, 2, ..., m-1, m-1, m, m-1, m-1, ..., 2, 2, 1, 1$. Now, a set of n pairs of distinct integers from $1, 2, ..., 2m$ that share the same sum is simply a set of n pairs of the $\left\lfloor \frac{(k-1)}{2} \right\rfloor$ or $\left\lfloor \frac{(4m-k+1)}{2} \right\rfloor$ pairs above. So the number of such sets is equal to

$$2\binom{1}{n} + 2\binom{2}{n} + ... + \binom{m}{n} + ... + 2\binom{2}{n} + 2\binom{1}{n}$$

$$= 4\left(\binom{1}{n} + \binom{2}{n} + ... + \binom{m-1}{n} + \binom{m}{n}\right) - 3\binom{m}{n}$$

$$= 4\binom{m+1}{n+1} - 3\binom{m}{n}$$

and this completes our proof.

20. 1st solution: We establish the following lemma.

Lemma: Let $f, g : R \to R$ be functions such that f is non constant and $af + bg$ is monotonic for all $a, b \in R$. Then there exists $c \in R$ such that $g - cf$ is a constant function.

Proof: Let s, t be two real numbers such that $f(s) \ne f(t)$. Let $u = \frac{(g(s)-g(t))}{(f(s)-f(t))}$. Let $h_1 = g - a_1 f$ for some $a_1 \in R$. Then h_1 is monotonic. But

$h_1(s) - h_1(t) = g(s) - g(t) - a_1(f(s) - f(t))$

$= (f(s) - f(t))(u - a_1)$

Since $f(s) - f(t) \neq 0$ is fixed, the monotonicity of h_1 depends only on the sign of $u - a_1$.

Since f is non-constant, there exist $x_1, x_2 \in R$ such that $f(x_1) \neq f(x_2)$.

Let $c = \dfrac{g(x_1) - g(x_2)}{f(x_1) - f(x_2)}$

and $h = g - cf$. Then $r = h(x_1) = h(x_2)$ and the monotonicity of $h_1 = g - a_1 f$, for each a_1, depends only on the sign of $c - a_1$. We claim that $h = g - cf$ is a constant function.

We prove our claim by contradiction. Suppose, on the contrary, that there exists $x_3 \in R$ such that $h(x_3) \neq r$. Since $f(x_1) \neq f(x_2)$, at least one of $f(x_1) \neq f(x_3)$ and $f(x_2) \neq f(x_3)$ is true. WLOG, suppose that $f(x_1) \neq f(x_3)$.

Let $c' = \dfrac{g(x_1) - g(x_3)}{f(x_1) - f(x_3)}$

Then the monotonicity of h_1 also depends only on the sign of $c' - a_1$. Since $h(x_3) \neq r = h(x_1)$,

$$c \neq \frac{g(x_1) - g(x_3)}{f(x_1) - f(x_3)} = c'$$

and hence $c - a_1 \neq c' - a_1$. So there exists some a_1 such that h_1 is both strictly increasing and decreasing, which is impossible. Therefore our assumption is false and h is a constant function.

Now we prove our main result. If f_1, f_2, f_3 are all constant functions, the result is trivial. WLOG, suppose that f_1 is non constant. For $a_3 = 0$, we apply the lemma to f_1 and f_2, so $f_2 = cf_1 + d$; for $a_2 = 0$, we apply the lemma to f_1 and f_3, so $f_3 = c'f_1 + d'$. Here c, c', d, d' are constants.

We have

$$(c'd - cd')f_1 + d'f_2 - df_3$$
$$= (c'd - cd')f_1 + d'(cf_1 + d) - d(c'f_1 + d') = 0$$

If $(c'd - cd', d', -d) \neq (0, 0, 0)$, then let

$$(c_1, c_2, c_3) = (c'd - cd', d', -d)$$

and we are done. Otherwise, $d = d' = 0$ and f_2, f_3 are constant multiples of f_1. Then the problem is again trivial.

2nd solution: Define the vector

$$v(x) = (f_1(x), f_2(x), f_3(x))$$

for $x \in R$. If the $v(x)$ span a proper subspace of R^3, we can find a vector (c_1, c_2, c_3) orthogonal to that subspace, and then $c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) = 0$ for all $x \in R$.

So suppose the $v(x)$ span all of R^3. Then there exist $x_1 < x_2 < x_3 \in R$ such that

$v_1 = v(x_1), v_2 = v(x_2), v_3 = v(x_3)$ are linearly independent, and so the 3×3 matrix A with $A_{ij} = f_j(x_i)$ has linearly independent, rows. But then A is invertible, and its columns also span R^3. This means we can find c_1, c_2, c_3 with $c_1 v_1 + c_2 v_2 + c_3 v_3 = (0, 1, 0)$ and the function $c_1 f_1 + c_2 f_2 + c_3 f_3$ is then not monotonic, a contradiction.

21. Label the disks as $d_1, d_2, ..., d_{12}$ in a clockwise order where disk d_k is the same as disk d_{k+12}. Let b denote the number of black disks. We consider the following cases.

(a) b is even. After each move, there will still be an even number of black disks. Therefore, it is impossible to do if there are an even number of black disks.

(b) n is odd and black disks and white disks are split in two groups. WLOG, let $d_1, ..., d_{2i+1}$ be black and $d_{2i+2}, ..., d_{12}$ be white. Then reversing the disks around $d_{2i}, d_{2i-2}, ..., d_2$ in that order will lead the position where $d_2, ..., d_{2i}$ are black and the others are white. We can then re-number the disks so d_2 is the first disk and repeat the process if necessary and we will end up with one black disk.

(c) n is odd and black disks and white disks are mixed. WLOG, let d_6 be a black disk and $d_i, ..., d_j$ be the block of black disks around d_6, *i.e.*, $i \leq 6 \leq j$ and both d_{i-1} and d_{j+1} are white. Let d_k be the black disk closest to d_j. Therefore, $d_{j+1}, ..., d_{k-1}$ are all white. Flip the disks around d_k then d_{k-1} now is black. Thus the distance between the block and the next black is getting smaller. Repeat the process if necessary. We will end up with case (b).

From the above, we see that it is possible to obtain the position where all disks but one are white, as long as there are an odd number of black disks to start with.

22. Note that for prime p, if $p^2 | n$ then $p | \phi(n)$ but $p \nmid n - 1$, contradiction. So we need only to show that

$n \neq pq$; $n \neq pqr$ for primes $p < q < r$.

First assume $n = pq$, so $(p-1)(q-1) | pq - 1$. Note that $q \geq 3$ implies that the left side is even, so the right is too and p, q are odd. But if $p = 3$, $q = 5$ then $\frac{(pq-1)}{(p-1)(q-1)} < 2$; the left side is decreasing in each variable and always > 1 so it cannot be an integer, contradiction.

Now let $n = pqr$. As before p, q, r are odd; if $p = 3$, $q = 7$, and $r = 11$ then

$$\frac{(pqr-1)}{(p-1)(q-1)(r-1)} < 2$$

and again the left side is decreasing and > 1; this eliminates all cases except where $p = 3$, $q = 5$. Then for $r = 7$, we have

$$\frac{(pqr-1)}{(p-1)(q-1)(r-1)} < 3$$

so the only integer value ever attainable is 2. Note that $\frac{(15r-1)}{8(r-1)} = 2$ gives $r = 15$ which is not a prime and we have eliminated all cases.

23. 1st solution: First we claim that at least one of bc and yz has its value less than 3. If $bc = 3$, then $b = 3$, $c = 1$, $a + b + c < 3a = abc$; if $bc > 3$, then $abc > 3a \geq a + b + c$. Thus for $bc \geq 3$, we have $abc > a + b + c$ and $3x \geq x + y + z = abc > a + b + c = xyz \Rightarrow 3 > yz$

This proves our claim. WLOG, suppose that $yz = 1$ or 2.

If $yz = 1$, then $y = z = 1$. We have

$$abc = x + y + z = x + 2 = xyz + 2 = a + b + c + 2.$$

If $c \geq 2$, then $bc \geq 4$ and $4a \leq abc = a + b + c + 2 \leq 4a$; thus $a = b = c = 2$. We obtain the solutions (2, 2, 2, 6, 1, 1) and (6, 1, 1, 2, 2, 2). If $c = 1$, then $ab = a + b + 3$. If $b \geq 3$, then $3a \leq ab = a + b + 3 \leq 3a \Rightarrow a = b = 3$

We obtain the solutions (3, 3, 1, 7, 1, 1) and (7, 1, 1, 3, 3, 1). If $b = 2$, we have $a = 5$ and obtain the solutions (5, 2, 1, 8, 1, 1) and (8, 1, 1, 5, 2, 1). If $b = 1$, we have $a = a + 4$, which is impossible.

If $yz = 2$, then $y = 2$, $z = 1$. We have

$$2abc = 2(x + y + z) = 2x + 6 = xyz + 6$$
$$= a + b + c + 6 \leq 3a + 6.$$

If $c \geq 2$, then $8a \leq 2abc \leq 3a + 6 \Rightarrow 5a < 6$, which contradicts the fact that $a \geq c$. Thus $c = 1$, and $2ab = a + b + 7$. If $b \geq 3$,

$6a \leq 2ab = a + b + 7 \Rightarrow a \leq \frac{b}{5} + \frac{7}{5}$ which contradicts the fact that $a \geq b$. If $b = 2$, then $4a = 2ab = a + 9$ and $a = 3$. We obtain the solution (3, 2, 1, 3, 2, 1). If $b = 1$, we have $a = 8$, repeating the solution (8, 1, 1, 5, 2, 1).

2nd solution: Let $A = (ab - 1)(c - 1)$,

$B = (a - 1)(b - 1)$

$X = (xy - 1)(z - 1)$, $Y = (x - 1)(y - 1)$.

Thus A, B, X, Y are non-negative integers such that

$$A + B + X + Y = 4.$$

Clearly, neither of c and z can be greater than 2; that would force either A or Y to be greater than 4, and contradict the fact that $A + B + X + Y = 4$. If $c = 2$, we have $a, b \geq 2$ and $A \geq 3$, $B \geq 1$. Thus $A = 3$, $B = 1$, $X = Y = 0$. This yields the solution (2, 2, 2, 6, 1, 1). Similarly, if $z = 2$, we have (6, 1, 1, 2, 2, 2) as a solution.

Now we suppose that $c = z = 1$. We have $A = X = 0$ and $B + Y = 4$. WLOG, suppose that $Y \leq B$, (*i.e.*, $Y = 0, 1, 2$.). If $Y = 0$, we have $B = (a - 1)(b - 1) = 4$. This leads to the solutions (5, 2, 1, 8, 1, 1) and (3,3,1,7,1,1). By symmetry, we also have the solutions (8, 1, 1, 5, 2, 1) and (7, 1, 1, 3, 3, 1). If $Y = 1$, then $x = y = 2$ and $B = (a - 1)(b - 1) = 3 \Rightarrow a = 4$, b = 2, but $a + b + c = 7 \neq xyz$.

If $Y = 2$, then $(x - 1)(y - 1) = (a - 1)(b - 1) = 2$ $\Rightarrow a = x = 3$, $b = y = 2$. We obtain (3, 2, 1, 3, 2, 1) as our last solution.

24. 1st solution: Let X_n, Y_n, Z_n be subsets of A_n that contain words that end up with letters a, b, c, respectively. For positive integer n, we have the following relations :

$$|A_n| = |X_n| + |Y_n| + |Z_n|;$$
$$|X_{n+1}| = |Y_n| + |Z_n|;$$
$$|Y_{n+1}| = |X_n| + |Z_n|;$$
$$|Z_{n+1}| = |X_n| + |Y_n| + |Z_n|;$$
$$|Z_{n+1}| = |A_n|,$$

which implies that

$$|A_{n+2}| = |X_{n+2}| + |Y_{n+2}| + |Z_{n+2}|$$
$$= 2|X_{n+1}| + 2|Y_{n+1}| + 3|Z_{n+1}| = 2|A_{n+1}| + |A_n|,$$

with $A_1 = 3$, $A_2 = 7$.

Let S_n be subsets of B_n that contain words that end up with letters *aa*, *bb*, *cc*. Let $T_n = B_n - S_n$, *i.e.*, the T_n contains those words in S_n that end up with *ab*, *ac*, *ba*, *bc*, *ca*, *cb*. For positive integer n, we have the following relations :

$$|B_n| = |S_n| + |T_n|; |B_{n+1}| = 3|S_n| + 2|T_n|;$$
$$|S_{n+1}| = |B_n|, \text{ which implies that}$$
$$|B_{n+2}| = |S_{n+2}| + |T_{n+2}| = 2|B_{n+1}| + |B_n|,$$

with $|B_2| = 3^2 = 9 = 3|A_1|$, $|B_3| = 3^3 - 3! = 21 = 3|A_2|$. Thus $|B_{n+1}| = 3|A_n|$.

2nd solution: Replace *a*, *b* and *c* by 1, 2 and 0 respectively, and words of length n by ordered n-tuples. Let S_n be the set of all ordered n-tuples in which each term is 0, 1, or 2. Consider the function $\Delta : S_{n+1} \to S_n$ defined as follows:

$$\Delta(x_1, ..., x_{n+1}) = (y_1, ..., y_n)$$

where $y_i \equiv x_{i+1} - x_i \pmod 3$. Observe that x_{i-1}, x_i, x_{i+1} are pairwise distinct if and only if $x_i - x_{i-1} = x_{i+1} - x_i \in \{1, 2\}$, so $(x_1, ..., x_{n+1})$ contains no three consecutive distinct elements if and only if $\Delta(x_1, ..., x_{n+1})$ contains no two consecutive 1's or 2's. Therefore $\Delta(B_{n+1}) = A_n$. Since $|\Delta^{-1}(\{y\})| = 3$ for $y \in S_{n-1}$, we have $|B_{n+1}| = 3|A_n|$.

25. The smallest such integer is $m^{m^{m+2}}$. First, suppose *A* and *B* are a partition of $\{m, m+1, ..., m^{m^{m+2}}\}$ into two subsets such that neither subset contains three numbers *a*, *b*, *c* such that $a^b = c$. Assume WLOG $m \in A$; then $m^m \in B$, $(m^m)^{(m^m)} = m^{m^{m+1}} \in A$, and $(m^{m^{m+1}})^m = m^{m^{m+2}} \in B$. Now consider the element m^{m+1}. If it is in set *A*, then we can take $a = m$, $b = m^{m+1}$, and $c = m^{m^{m+1}}$, a contradiction. If it is in set *B*, take $a = m^m$, $b = m^{m+1}$, and $c = m^{m^{m+2}}$, again a contradiction. So $n = m^{m^{m+2}}$ satisfies the desired property.

Now we exhibit neither subset contains three numbers *a*, *b*, *c* such that $a^b = c$. Take

$$A = \{m, m+1, ..., m^m - 1\} \cup \{m^{m^{m+1}}, ..., m^{m^{m+2}} - 1\}$$

and $B = \{m^m, ..., m^{m^{m+1}} - 1\}$. Then for any $a, b \in B$, $a^b \geq (m^m)^{(m^m)} = m^{m^{m+1}}$ so $a^b \notin B$.

Also, there are no $a, b, \in A$ such that $a^b \in A$ because of the inequalities $m^m > m^m - 1$,

$$(m^m - 1)^{(m^m - 1)} < (m^m)^{(m^m)} = m^{m^{m+1}},$$
$$(m^{m^{m+1}})^m > m^{m^{m+2}} - 1$$

Thus $m^{m^{m+2}}$ is the smallest such *n*.

26. The solutions are $(x, n) = (4, 1)$ and $(11, 1)$.

If $n = 1$, we need $x + 3 = x + 2 + 1 | x^2 + 4 + 1 = x^2 + 5 = (x + 3)(x - 3) + 14$, so $x + 3$ divides 14 and $x = 4$ or 11. Suppose $n \geq 2$. For $x \in \{1, 2, 3\}$ we have

$$1 + 2^n + 1 < 1 + 2^{n+1} + 1 < 2(1 + 2^n + 1),$$
$$2^n + 2^n + 1 < 2^{n+1} + 2^{n+1} + 1 < 2(2^n + 2^n + 1),$$
$$2(3^n + 2^n + 1) < 3^{n+1} + 2^{n+1} + 1 < 3(3^n + 2^n + 1),$$

so $x^n + 2^n + 1$ does not divide $x^{n+1} + 2^{n+1} + 1$. For $x \geq 4$, $x^n = \frac{x^n}{2} + \frac{x^n}{2} \geq \frac{2^{2n}}{2} + \frac{x^2}{2}$, so

$$(2^n + 1)x \leq \frac{((2^n + 1)^2 + x^2)}{2}$$
$$= \frac{(2^{2n} + 2^{n+1} + 1 + x^2)}{2} < 2^{n+1} + x^n + 2^n + 2.$$

Therefore

$$(x - 1)(x^n + 2^n + 1) = x^{n+1} + 2^n x + x - x^n - 2^n - 1$$
$$< x^{n+1} + 2^{n+1} + 1 < x(x^n + 2^n + 1)$$

again $x^n + 2^n + 1$ does not divide $x^{n+1} + 2^{n+1} + 1$. So the only solutions are (4, 1) and (11, 1).

27. The only such triangle is a 3-4-5 triangle. Let *r* be the inradius of the triangle; *a*, *b*, *c* its side lengths; and *s* its semiperimeter. The smallest and largest of the four integers have the same sum as the middle two integers, so WLOG, we suppose that $2r + a = b + c$. Then $2r = b + c - a$ and $r = s - a$. Let *I* be the incenter of the triangle, and *E* and *F* be the points of tangency of the incircle with *CA* and *AB*. $IE = EA = AF = FI = s - a$. Thus *IEAF* is a square and $\angle A = 90°$. So *a* must be the largest, integer, $2r$ the smallest, and *a*, *b*, *c* are consecutive integers. The only such right triangle is a 3-4-5 triangle, in which case $2r = 2$ indeed works. ■■

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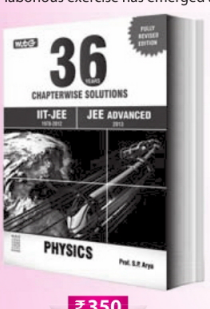

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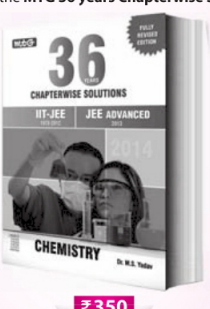

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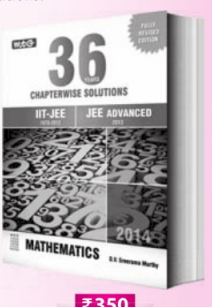

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ENRICH YOUR CONCEPTS ON SETS

Based on NCERT Pattern

XI

SETS

Definition : A well defined collection of objects is called a set. The objects are called members or elements of the set. Sets are usually denoted by capital letters and their members are denoted by small letters.

- If a is an element of a set A, then we write '$a \in A$'. We say that a belongs to A.
- If b is not an element of a set A, then we write '$b \notin A$'. We say that b does not belongs to A.

REPRESENTATION OF SETS

S.No.	Method	Description	Example
1.	Roster or Tabular Form	All the members of the set are listed, the elements are being separated by commas and are enclosed within braces { }.	• The set of even numbers less than 21 is represented by {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}
2.	Set-Builder Form	One or more variables (say x, y, etc.) are taken to represent all the properties possessed by every element of the set.	• The set of all real numbers greater than –1 and less than or equal to 1 can be written as $\{x \in R : -1 < x \leq 1\}$ • The set of rational numbers can be written as $Q = \{x : x = \frac{m}{n}$; where m and n are integers and $n \neq 0\}$.

TYPES OF SETS

S. No.	Types	Definition	Examples
1.	Empty Set	A set is said to an be empty or null or void set if it has no element and is denoted by ϕ. In Roster method, ϕ is denoted by { }.	The set A is given by $A = \{x : x$ is an even prime number greater than 2$\}$ is an empty set because 2 is the only even prime number.
2.	Singleton Set	A set consisting of a single element is called a singleton set.	The set A is given by $A = \{x : x \in N$ and $x^2 = 49\}$ is a singleton set equal to {7}.
3.	Finite Set	A set whose elements can be counted *i.e.,* a set which has only finite number of elements is called finite set.	The set A is given by $A = \{x : x \in N$ and $1 \leq x \leq 25\}$ is finite because elements can be counted.
4.	Infinite Set	A set which is not finite is called infinite set.	The set A is given by $A = \{x : x \in R$ and $0 < x < 1\}$ is infinite because there are infinitely many real numbers greater than 0 and less than 1.

5.	Equivalent Sets	Two finite sets A and B are said to be equivalent if number of elements in both the sets is same *i.e.*, $n(A) = n(B)$.	Let A and B be two finite sets. $A = \{1, 2, 3,\}$ & $B = \{a, b, c\}$ are equivalent sets as both contain 3 elements.
6.	Equal Sets	Two sets A and B are said to be equal if every element of A is a member of B and every element of B is a member of A.	Let A and B be two sets. $A = \{0, 1\}$ and $B = \{x : x \in Z, x^2 - x = 0\}$ are equal sets as roster form of set B is $\{0, 1\}$.

Note : Equal sets are equivalent but equivalent sets need not be equal.

SOME SPECIAL TYPES OF SETS

- **Subset :** A set A is said to be the subset of set B if every member of set A is also the member of set B. It is denoted as $A \subseteq B$.
  If A is not a subset of B, then we write $A \nsubseteq B$
  Note :
  (i) Empty set is the subset of every set.
  (ii) Every set is the subset of itself.
  (iii) Total number of subsets of a set having n elements is 2^n.
- **Superset :** If $A \subseteq B$, then B is called the superset of A, and we write, $B \supseteq A$.
- **Proper Subset :** If $A \subseteq B$ and $A \neq B$, then A is called proper subset of B and we write $A \subset B$.
- **Power Set :** Set of all the subsets of a set is called power set of the set. Power set of set A is denoted by $P(A)$.
  i.e., $P(A) = \{S : S \subseteq A\}$

Subsets of the set R of Real Numbers :

Following sets are important subsets of the set R of all real numbers.

(i) The set of all natural numbers
$N = \{1, 2, 3, 4, 5, 6, ...\}$

(ii) The set of all integers
$Z = \{....., -3, -2, -1, 0, 1, 2, 3, ...\}$

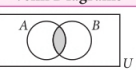
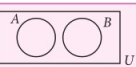
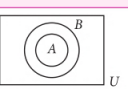
(iii) The set of all rational numbers
$Q = \{x : x = \frac{m}{n}, m, n \in Z, n \neq 0\}$

(iv) The set of all irrational numbers
$T = \{x : x \in R \ \& \ x \notin Q\}$
Hence $N \subset Z \subset Q \subset R$, $T \subset R$, $N \not\subset T$

- **Universal Set :** If all sets under consideration are subset of a larger set, then this larger set is called universal set, denoted by U.
  Note : Universal set is not a fixed set, it varies from situation to situation.

VENN DIAGRAMS

- In Venn diagrams the universal set U is represented by using rectangle and its subsets are represented by using circle (a closed curve).

Cases	Venn Diagrams
If two sets A and B are joint but $A \neq B$. The shaded part represents the common points.	
If A and B are disjoint.	
If A is subset of B. (A is a proper subset of B)	

OPERATION ON SETS

- **Union of two Sets :** Let A and B be two sets. The union of A and B is the set of all those elements which belongs to either A or B or both. The union of two sets A and B is denoted by $A \cup B$ (read as "A union B")
  Thus $A \cup B = \{x : x \in A \text{ or } x \in B\}$
  $\therefore \quad x \in A \cup B \Leftrightarrow x \in A \text{ or } x \in B$
  If $x \notin A \cup B$, then we say $x \notin A$ and $x \notin B$.

denotes $A \cup B$

- **Some Properties of Union**
  (i) $A \cup B = B \cup A$ (Commutative law)
  (ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

(iii) $A \cup \phi = A$ (Law of identity element, ϕ is the identity of $\cup$)

(iv) $A \cup A = A$ (Idempotent law)

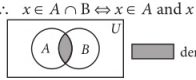
(v) $U \cup A = U$ (Law of U)

- **Intersection of two sets :** Let A and B be two sets. The intersection of A and B is the set of all those elements which is present in both sets A and B.

  The intersection of two sets A and B is denoted by $A \cap B$ (read as "A intersection B")

  Thus $A \cap B = \{x : x \in A \text{ and } x \in B\}$

  $\therefore \quad x \in A \cap B \Leftrightarrow x \in A$ and $x \in B$.



denotes $A \cap B$

- **Some Properties of Intersection :**

(i) $A \cap B = B \cap A$ (Commutative law)

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)

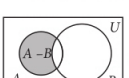
(iii) $\phi \cap A = \phi$, $U \cap A = A$ (Law of ϕ and U)

(iv) $A \cap A = A$ (Idempotent law)

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) *i.e.*, $\cap$ distributes over $\cup$

- **Difference of two sets :** Let A and B be two sets, then the difference from set A to set B, is the set of all those elements which belongs to A but does not belongs to B. We denote it by $A - B$ & define it as $A - B = A - (A \cap B)$

$= \{x : x \in A \text{ but } x \notin B\}$



$= A - B = A - (A \cap B) =$ only A.

$= B - A = B - (A \cap B) =$ only B.

The sets $A - B$, $A \cap B$ and $B - A$ are mutually disjoint sets *i.e.* the intersection of any of these two sets is the null set.

- **Symmetric Difference :** Let A and B be two sets. The symmetric difference of sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$.

$\therefore \quad A \Delta B = (A - B) \cup (B - A) = \{x : x \notin A \cap B\}$

denotes $(A - B) \cup (B - A)$

COMPLEMENT OF A SET

If U be the universal set and let A be a subset of U *i.e.*, $A \subset U$, then complement of A with respect to U is the set of elements which belongs to U but not present in A. We denote it as A^c or A' or $\overline{A}$ or $U - A$ *i.e.*, $A' = \{x : x \in U \text{ but } x \notin A\}$

$\therefore \quad x \notin A' \Leftrightarrow x \in A$

denotes A'

- **Some Properties of Complement of Sets**

1. Complement laws :

(i) $A \cup A' = U$

(ii) $A \cap A' = \phi$

2. De Morgan's law :

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

3. Law of double complementation : $(A')' = A$

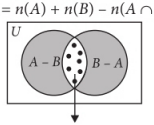
4. Laws of empty set and universal set :

$\phi' = U$ and $U' = \phi$

PRACTICAL PROBLEMS ON UNION AND INTERSECTION

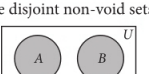
If A, B, C are finite sets and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



(ii) $n(A \cup B) = n(A) + n(B)$

$\Leftrightarrow A, B$ are disjoint non-void sets



SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS OF SETS

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A - B) = n(A) - n(A \cap B)$

(iii) $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

(iv) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

VERY SHORT ANSWER TYPE (1 MARK)

1. Write the set $\{x : x$ is a positive integer and $x^2 < 40\}$ in the roster form.

2. Write the following sets in the set builder form :
(i) $\{-1, 1\}$ **(ii)** {B, E, N, G, I}

3. Let $A = \{1\}$, B {1, 2, 3} and $C = \{1, 4\}$ be three sets, then which of them are subsets?

4. Write all the possible subsets of {1, 2}.

5. Let $A = \{x : x$ is a letter of the word "LITTLE"$\}$, then find $P(A)$.

SHORT ANSWER TYPE (4 MARKS)

6. Write each of the following sets as intervals:
(i) $\{x \in R : -1 < x \leq 2\}$
(ii) $\left\{x \in R : \frac{3}{2} < 2 + 3x < 5\right\}$

7. Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$.

8. A market research group conducted a survey of 2000 consumers and reported that 1720 consumers liked product P_1 and 1450 consumers liked product P_2. What is the least number that must have liked both the products?

9. Show that
(i) $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$
(ii) $(A \cup B) - (A - B) = B$.

10. Let $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$. Find $V - B$ and $B - V$ and represent by venn diagrams.

LONG ANSWER TYPE (6 MARKS)

11. A survey of 500 television viewers produced the following information: 285 watch football, 195 watch hockey, 115 watch basketball, 45 watch football and basketball, 70 watch football and hockey, 50 watch hockey and basketball and 50 do not watch any of the three games.
(i) How many watch all the three games?
(ii) How many watch exactly one of the three games?

12. In a survey of 100 persons it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazines A and B, 10 read magazines A and C, 5 read magazines B and C and 3 read all the three magazines. Find:
(i) How many read none of three magazines?
(ii) How many read magazine C only?

13. For any sets A and B, prove that $P(A \cap B) = P(A) \cap P(B)$, where $P(X)$ represent the power set of set X.

14. Using laws of set algebra, show that
(i) $(A \cup B) \cap (A \cup B') = A$
(ii) $(A - B) \cup (A \cap B) = A$
(iii) If $A \cap B' = \phi$, show that $A \subset B$
(iv) If $A' \cup B = U$, show that $A \subset B$.

15. In a survey of 100 students, the number of students studying the various languages were found to be: English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find:
(i) How many students were studying Hindi?
(ii) How many students were studying English and Hindi?

SOLUTIONS

1. The required numbers are 1, 2, 3, 4, 5, 6. So, the given set in the roster form is {1, 2, 3, 4, 5, 6}.

2. **(i)** Set builder form of $\{-1, 1\}$ is $\{x : x$ is a root of the equation $x^2 = 1\}$.
(ii) Set builder form of {B, E, N, G, I} is $\{x : x$ is a letter in the word 'BEGIN'$\}$.

3. $1 \in A$ and $1 \in B \Rightarrow A \subseteq B$
$1 \in A$ and $1 \in C \Rightarrow A \subseteq C$

4. The possible subsets of A are ϕ, {1}, {2}, {1, 2}

5. Roster form of set A is {L, I, T, E}
Subsets of $A = \phi$, {L}, {I}, {T}, {E}, {L, I}, {L, T},

{L, E}, {I, E}, {I, T}, {T, E}, {L, I, T}, {L, I, E}, {I, T, E}, {L, T, E}, {L, I, T, E}

$P(A) = \{\phi, \{L\}, \{I\}, \{T\}, \{E\}, \{L, I\}, \{L, T\}, \{L, E\}, \{I, E\}, \{I, T\}, \{T, E\}, \{L, I, T\}, \{L, I, E\}, \{I, T, E\}, \{L, T, E\}, \{L, I, T, E\}\}$

6. (i) The given set

$= \{x \in R : -1 < x \leq 2\} = (-1, 2]$

(ii) The given set

$$= \left\{x \in R : \frac{3}{2} < 2 + 3x < 5\right\}$$

$$= \left\{x \in R : \frac{3}{2} - 2 < 3x < 5 - 2\right\}$$

$$= \left\{x \in R : -\frac{1}{2} < 3x < 3\right\}$$

$$= \left\{x \in R : -\frac{1}{6} < x < 1\right\} = \left(-\frac{1}{6}, 1\right)$$

7. We have $A \cup B = A \cup C$

$\Rightarrow (A \cup B) \cap C = (A \cup C) \cap C$

$\Rightarrow (A \cap C) \cup (B \cap C) = C$

$[\because (A \cup C) \cap C = C]$

$\Rightarrow (A \cap B) \cup (B \cap C) = C$ ...(i)

$[\because A \cap C = A \cap B]$

Again, $A \cup B = A \cup C$

$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B$

$\Rightarrow B = (A \cap B) \cup (C \cap B)$

$[\because (A \cup B) \cap B = B]$

$\Rightarrow B = (A \cap B) \cup (B \cap C)$ ...(ii)

From (i) and (ii), we get $B = C$.

8. Let U be the set of all consumers who were questioned, A be the set of consumers who liked product P_1 and B be the set of consumers who liked the product P_2.

It is given that

$n(U) = 2000, n(A) = 1720, n(B) = 1450$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 1720 + 1450 - n(A \cap B)$

$= 3170 - n(A \cap B)$

Since, $A \cup B \subseteq U$

$\therefore \quad n(A \cup B) \leq n(U)$

$\Rightarrow 3170 - n(A \cap B) \leq 2000$

$\Rightarrow 3170 - 2000 \leq n(A \cap B)$

$\Rightarrow n(A \cap B) \geq 1170$

Thus, the least value of $n(A \cap B)$ is 1170.

Hence, the least number of consumers who liked both the products is 1170.

9. **(i)** $(A \cup B) - (A \cap B) = (A \cup B) \cap (A \cap B)^c$

$(\because \quad A - B = A \cap B^c)$

$= (A \cup B) \cap (A^c \cup B^c)$

$= \{(A \cup B) \cap A^c\} \cup \{(A \cup B) \cap B^c\}$

$= \{(A \cap A^c) \cup (B \cap A^c)\} \cup$

$\{(A \cap B^c) \cup (B \cap B^c)\}$

$= \{\phi \cup (B \cap A^c)\} \cup \{(A \cap B^c) \cup \phi\}$

$= (B \cap A^c) \cup (A \cap B^c)$

$= (B - A) \cup (A - B) = (A - B) \cup (B - A).$

(ii) $(A \cup B) - (A - B) = (A \cup B) \cap (A - B)'$

$= (A \cup B) \cap (A \cap B')'$

$= (A \cup B) \cap (A' \cup B)$

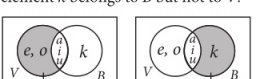
$= (A \cap A') \cup B = \phi \cup B = B.$

10. We have,

$V - B = \{a, e, i, o, u\} - \{a, i, k, u\} = \{e, o\}$,

since the elements e, o belong to V but not to B and

$B - V = \{a, i, k, u\} - \{a, e, i, o, u\} = \{k\}$, since the element k belongs to B but not to V.



We note that $V - B \neq B - V$.

11. N = Total number of television viewers = 500, Let F, H, B denote the set of the viewers who watch football, hockey and basketball respectively.

$n(F) = 285, n(H) = 195, n(B) = 115,$

$n(F \cap B) = 45, n(F \cap H) = 70, n(H \cap B) = 50,$

$n(F' \cap H' \cap B') = 50,$

Now, $n(F' \cap H' \cap B') = 50$

$\Rightarrow \quad n(F \cup H \cup B)' = 50$

$\Rightarrow \quad N - n(F \cup H \cup B) = 50$

$\Rightarrow \quad 500 - [n(F) + n(H) + n(B) - n(F \cap H) - n(F \cap B) - n(H \cap B) + n(F \cap H \cap B)] = 50$

$\Rightarrow \quad n(F \cap H \cap B) = 500 - 285 - 195 - 115 + 70 + 45 + 50 - 50 = 20.$

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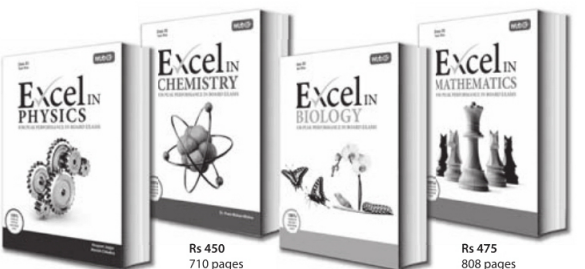

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(i) Required number = $n(F \cap H \cap B) = 20$

(ii) Required number = $n(F \cap H' \cap B') +$
$n(F' \cap H' \cap B) + n(F' \cap H \cap B')$
$= n(F) + n(H) + n(B) - 2[n(F \cap H) + n(H \cap B) + n(B \cap F)] + 3n(F \cap H \cap B)$
$= 285 + 195 + 115 - 2[70 + 50 + 45] + 3[20]$
$= 595 - 330 + 60 = 325$

12. Let A, B, C be the set of persons who read magazine A, B and C respectively.
$n(A) = 28$, $n(B) = 30$, $n(C) = 42$, $n(A \cap B) = 8$, $n(A \cap C) = 10$, $n(B \cap C) = 5$, $n(A \cap B \cap C) = 3$

(i) $n(A' \cap B' \cap C') = n(A \cup B \cup C)'$
$= n(U) - n(A \cup B \cup C)$
$= n(U) - [n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)]$
$= 100 - [28 + 30 + 42 - 8 - 10 - 5 + 3]$
$= 100 - [103 - 23] = 20$

(ii) $n(A' \cap B' \cap C) = n[C \cap (A \cup B)']$
$= n(C) - n[C \cap (A \cup B)]$
$= n(C) - n[(C \cap A) \cup (C \cap B)]$
$= n(C) - [n(C \cap A) + n(C \cap B) - n(A \cap B \cap C)]$
$= 42 - [10 + 5 - 3] = 42 - 12 = 30$

13. Let $X \in P(A \cap B)$. Then
$X \in P(A \cap B) \Rightarrow X \subseteq A \cap B$
$\Rightarrow X \subseteq A$ and $X \subseteq B$
$\Rightarrow X \in P(A)$ and $X \in P(B)$
$\Rightarrow X \in P(A) \cap P(B)$.
$\therefore \quad P(A \cap B) \subseteq P(A) \cap P(B)$ …(i)
Again, let $Y \in P(A) \cap P(B)$. Then,
$Y \in P(A) \cap P(B) \Rightarrow Y \in P(A)$ and $Y \in P(B)$
$\Rightarrow Y \subseteq A$ and $Y \subseteq B$
$\Rightarrow Y \subseteq A \cap B$
$\Rightarrow Y \in P(A \cap B)$.
$\therefore \quad P(A) \cap P(B) \subseteq P(A \cap B)$. …(ii)
From (i) and (ii), we get
$P(A \cap B) = P(A) \cap P(B)$.

14. **(i)** $(A \cup B) \cap (A \cup B') = A \cup (B \cap B')$ (Distributive law)
$= A \cup \phi = A$.

(ii) $(A - B) \cup (A \cap B) = (A \cap B') \cup (A \cap B)$
$= A \cap (B' \cup B)$
$= A \cap X$, X being the universal set.
$= A$

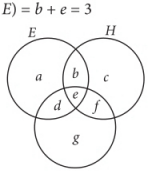
(iii) Now, $A = A \cap Y$, where Y is the universal set
$= A \cap (B \cup B') = (A \cap B) \cup (A \cap B')$
$= (A \cap B) \cup \phi \qquad (\because A \cap B' = \phi)$
$= A \cap B$
Hence, $A = A \cap B \Rightarrow A \subset B$.

(iv) Now, $B = B \cup \phi = B \cup (A \cap A')$
$= (B \cup A) \cap (B \cup A')$
$= (B \cup A) \cap U = B \cup A$
Hence, $B = B \cup A$, *i.e.*, $B = A \cup B \Rightarrow B \supset A$
$\Rightarrow A \subset B$

15. Let E, H, S be the number of students studying English, Hindi and Sanskrit respectively.
We have, $a = 18$, $a + d = 23$, $d + e = 8$,
$a + b + d + e = 26$, $d + e + f + g = 48$, $e + f = 8$,
$a + b + c + d + e + f + g = 100 - 24 = 76$
$\therefore \quad a = 18$, $b = 0$, $c = 10$, $d = 5$, $e = 3$, $f = 5$ and $g = 35$

(i) $n(H) = b + c + e + f = 18$

(ii) $n(H \cap E) = b + e = 3$



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SOLVED PAPER 2014

Kerala PET

1. If the operation $\oplus$ is defined by $a \oplus b = a^2 + b^2$ for all real numbers a and b, then $(2 \oplus 3) \oplus 4 =$
(a) 120 (b) 185 (c) 175 (d) 129
(e) 312

2. The number of students who take both the subjects mathematics and chemistry is 30. This represents 10% of the enrolment in mathematics and 12% of the enrolment in chemistry. How many students take at least one of these two subjects?
(a) 520 (b) 490 (c) 560 (d) 480
(e) 540

3. Let $f(x) = |x - 2|$, where x is a real number. Which one of the following is true?
(a) f is periodic
(b) $f(x + y) = f(x) + f(y)$
(c) f is an odd function
(d) f is not a 1-1 function
(e) f is an even function

4. If $A = \{1, 3, 5, 7\}$ and $B = (1, 2, 3, 4, 5, 6, 7, 8\}$, then the number of one-to-one functions from A into B is
(a) 1340 (b) 1860 (c) 1430 (d) 1880
(e) 1680

5. The range of the function $f(x) = x^2 + 2x + 2$ is
(a) $(1, \infty)$ (b) $(2, \infty)$ (c) $(0, \infty)$ (d) $(1, \infty)$
(e) $(-\infty, \infty)$

6. If $f(x) = \sqrt{x}$ and $g(x) = 2x - 3$, then $(fog)(x)$ is
(a) $(-\infty, -3)$ (b) $\left(-\infty, -\frac{3}{2}\right)$
(c) $\left[-\frac{3}{2}, 0\right]$ (d) $\left[0, \frac{3}{2}\right]$
(e) $\left[\frac{3}{2}, \infty\right)$

7. If $z = \frac{(\sqrt{3} + i)^3 (3i + 4)^2}{(8 + 6i)^2}$, then $|z|$ is equal to
(a) 8 (b) 2 (c) 5 (d) 4
(e) 10

8. Let $w \neq \pm 1$ be a complex number. If $|w| = 1$ and $z = \frac{w - 1}{w + 1}$, then $\mathrm{Re}(z)$ is equal to
(a) 1 (b) $\frac{1}{|w + 1|}$
(c) $\mathrm{Re}(w)$ (d) 0
(e) $w + \bar{w}$

9. If $z = e^{2\pi i/3}$, then $1 + z + 3z^2 + 2z^3 + 2z^4 + 3z^5$ is equal to
(a) $-3e^{\pi i/3}$ (b) $3e^{\pi i/3}$
(c) $3e^{2\pi i/3}$ (d) $-3e^{2\pi i/3}$
(e) 0

10. If $z_1 = 2\sqrt{2}(1 + i)$ and $z_2 = 1 + i\sqrt{3}$, then $z_1^2 z_2^3$ is equal to
(a) $128i$ (b) $64i$ (c) $-64i$ (d) $-128i$
(e) 256

11. If the complex numbers z_1, z_2 and z_3 denote the vertices of an isosceles triangle, right angled at z_1, then $(z_1 - z_2)^2 + (z_1 - z_3)^2$ is equal to
(a) 0 (b) $(z_2 + z_3)^2$
(c) 2 (d) 3
(e) $(z_2 - z_3)^2$

12. If the roots of $x^3 - ax + b = 0$ are two consecutive odd integers, then $a^2 - 4b$ is
(a) 3 (b) 4 (c) 5 (d) 6
(e) 7

13. If α and β are the roots of $x^2 - ax + b^2 = 0$, then $\alpha^2 + \beta^2$ is equal to

(a) $a^2 + 2b^2$ (b) $a^2 - 2b^2$
(c) $a^2 - 2b$ (d) $a^2 + 2b$
(e) $a^2 - b^2$

14. If α and β are the roots of the equation $x^2 + 3x - 4 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to

(a) $\frac{-3}{4}$ (b) $\frac{3}{4}$ (c) $\frac{-4}{3}$ (d) $\frac{4}{3}$
(e) $\frac{3}{2}$

15. The value of x such that $3^{2x} - 2(3^{x+2}) + 81 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

16. If the roots of the equation $x^2 + 2bx + c = 0$ are α and β, then $b^2 - c =$

(a) $\frac{(\alpha - \beta)^2}{4}$ (b) $(\alpha + \beta)^2 - \alpha\beta$
(c) $(\alpha + \beta)^2 + \alpha\beta$ (d) $\frac{(\alpha - \beta)^2}{2} + \alpha\beta$
(e) $\frac{(\alpha + \beta)^2}{2} + \alpha\beta$

17. The equation whose roots are the squares of the roots of the equation $2x^2 + 3x + 1 = 0$ is

(a) $4x^2 + 5x + 1 = 0$ (b) $4x^2 - x + 1 = 0$
(c) $4x^2 - 5x - 1 = 0$ (d) $4x^2 - 5x + 1 = 0$
(e) $4x^2 + 5x - 1 = 0$

18. The sum of the series $\sum_{n=8}^{17} \frac{1}{(n+2)(n+3)}$ is equal to

(a) $\frac{1}{17}$ (b) $\frac{1}{18}$ (c) $\frac{1}{19}$ (d) $\frac{1}{20}$
(e) $\frac{1}{21}$

19. If two positive numbers are in the ratio $3 + 2\sqrt{2} : 3 - 2\sqrt{2}$, then the ratio between their A.M. and G.M. is

(a) 6 : 1 (b) 3 : 2 (c) 2 : 1 (d) 3 : 1
(e) 1 : 6

20. Let $x_1, x_2, \ldots\ldots, x_n$ be in an A.P. If $x_1 + x_4 + x_9 + x_{11} + x_{20} + x_{22} + x_{27} + x_{30} = 272$, then $x_1 + x_2 + x_3 + \ldots\ldots\ldots + x_{30}$ is equal to

(a) 1020 (b) 1200 (c) 716 (d) 2720
(e) 2072

21. If the second and fifth terms of a G.P. are 24 and 3 respectively, then the sum of first six terms is

(a) 181 (b) $\frac{181}{2}$ (c) 189 (d) $\frac{189}{2}$
(e) 191

22. If the sum of first 75 terms of an A.P. is 2625, then the 38th term of the A.P. is

(a) 39 (b) 37 (c) 36 (d) 38
(e) 35

23. If -5, k, -1 are in A.P., then the value of k is equal to

(a) -5 (b) -3 (c) -1 (d) 3
(e) 5

24. Let T_n denote the number of triangles which can be formed by using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 36$, then n is equal to

(a) 2 (b) 5 (c) 6 (d) 8
(e) 9

25. The middle term in the expansion of $\left(\frac{10}{x} + \frac{x}{10}\right)^{10}$ is

(a) ${}^{10}C_5$ (b) ${}^{10}C_6$
(c) ${}^{10}C_5 \frac{1}{x^{10}}$ (d) ${}^{10}C_5 x^{10}$
(e) ${}^{10}C_5 10^{10}$

26. The coefficient of x^{49} in the product $(x-1)(x-2)(x-3) \ldots\ldots (x-50)$ is

(a) -2250 (b) -1275 (c) 1275 (d) 2250
(e) -49

27. The sum of the coefficients in the binomial expansion of $\left(\frac{1}{x} + 2x\right)^6$ is equal to

(a) 1024 (b) 729 (c) 243 (d) 512
(e) 64

28. The value of ${}^2P_1 + {}^3P_1 + \ldots\ldots + {}^nP_1$ is equal to

(a) $\frac{n^2 - n + 2}{2}$ (b) $\frac{n^2 + n + 2}{2}$
(c) $\frac{n^2 + n - 1}{2}$ (d) $\frac{n^2 - n - 1}{2}$

(e) $\frac{n^2+n-2}{2}$

29. How many four digit numbers *abcd* exist such that *a* is odd, *b* is divisible by 3, *c* is even and *d* is prime?
(a) 380 (b) 360 (c) 400 (d) 520
(e) 480

30. If $a_1, a_2, a_3, \ldots\ldots$ are in A.P., then the value of $\begin{vmatrix} a_1 & a_2 & 1 \\ a_2 & a_3 & 1 \\ a_3 & a_4 & 1 \end{vmatrix}$ is equal to
(a) $a_4 - a_1$ (b) $\frac{a_1+a_4}{2}$
(c) 1 (d) $\frac{a_2+a_3}{2}$
(e) 0

31. If $\begin{vmatrix} 2a & x_1 & y_1 \\ 2b & x_2 & y_2 \\ 2c & x_3 & y_3 \end{vmatrix} = \frac{abc}{2} \neq 0$, then the area of the triangle whose vertices are $\left(\frac{x_1}{a}, \frac{y_1}{a}\right), \left(\frac{x_2}{b}, \frac{y_2}{b}\right)$ $\left(\frac{x_3}{c}, \frac{y_3}{c}\right)$ is
(a) $\frac{1}{4}abc$ (b) $\frac{1}{8}abc$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$
(c) $\frac{1}{12}$

32. The system of linear equations $3x + y - z = 2$, $x - z = 1$ and $2x + 2y + az = 5$ has unique solution when
(a) $a \neq 3$ (b) $a \neq 4$
(c) $a \neq 5$ (d) $a \neq 2$
(e) $a \neq 1$

33. If $A = \begin{bmatrix} 2-k & 2 \\ 1 & 3-k \end{bmatrix}$ is a singular matrix, then the value of $5k - k^2$ is equal to
(a) 0 (b) 6 (c) -6 (d) -4
(e) 4

34. If *a, b, c* are non-zero and different from 1, then the value of

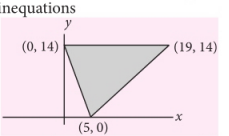
$$\begin{vmatrix} \log_a 1 & \log_a b & \log_a c \\ \log_a\left(\frac{1}{b}\right) & \log_b 1 & \log_a\left(\frac{1}{c}\right) \\ \log_a\left(\frac{1}{c}\right) & \log_a c & \log_c 1 \end{vmatrix} \text{ is}$$

(a) 0 (b) $1 + \log_a(a + b + c)$
(c) $\log_a(ab + bc + ca)$ (d) 1
(e) $\log_a(a + b + c)$

35. The number of solutions for the system of equations $2x + y = 4$, $3x + 2y = 2$ and $x + y = -2$ is
(a) 1 (b) 2
(c) 3 (d) infinitely many
(e) 0

36. The number of solutions of the inequation $|x - 2| + |x + 2| < 4$ is
(a) 1 (b) 2 (c) 4 (d) 0
(e) infinite

37. The shaded region shown in the figure is given by the inequations



(a) $14x + 5y \geq 70$, $y \leq 14$ and $x - y \geq 5$
(b) $14x + 5y \leq 70$, $y \leq 14$ and $x - y \geq 5$
(c) $14x + 5y \geq 70$, $y \geq 14$ and $x - y \geq 5$
(d) $14x + 5y \geq 70$, $y \geq 14$ and $x - y \leq 5$
(e) $14x + 5y \geq 70$, $y \leq 14$ and $x - y \leq 5$

38. Let *p, q* and *r* be any three logical statements. Which one of the following is true?
(a) $\sim [p \wedge (\sim q)] \equiv (\sim p) \wedge q$
(b) $\sim (p \vee q) \wedge (\sim r) \equiv (\sim p) \vee (\sim q) \vee (\sim r)$
(c) $\sim [p \vee (\sim q)] \equiv (\sim p) \wedge q$
(d) $\sim [p \wedge (\sim q)] \equiv (\sim p) \wedge \sim q$
(e) $\sim [p \wedge (\sim q)] \equiv p \wedge q$

39. The truth values of *p, q* and *r* for which $(p \wedge q) \vee (\sim r)$ has truth value F are respectively
(a) F, T, F (b) F, F, F (c) T, T, T (d) T, F, F
(e) F, F, T

40. $\sim [(\sim p) \wedge q]$ is logically equivalent to
(a) $\sim (p \vee q)$ (b) $\sim [p \wedge (\sim q)]$
(c) $p \wedge (\sim q)$ (d) $p \vee (\sim q)$
(e) $(\sim p) \vee (\sim q)$

41. Let $\theta \in \left[0, \frac{\pi}{2}\right]$. Which one of the following is true?
(a) $\sin^2\theta > \cos^2\theta$ (b) $\sin^2\theta < \cos^2\theta$
(c) $\sin\theta > \cos\theta$ (d) $\cos\theta > \sin\theta$
(e) $\sin\theta + \cos\theta \leq \sqrt{2}$

42. The value of $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) + \sin^{-1}\left(\frac{1}{3}\right)$ is equal to
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$
(e) 0

43. If $ab < 1$ and
$$\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) + \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = 2\tan^{-1}x,$$
then x is equal to
(a) $\frac{a}{1+ab}$ (b) $\frac{a}{1-ab}$
(c) $\frac{a-b}{1+ab}$ (d) $\frac{a+b}{1+ab}$
(e) $\frac{a+b}{1-ab}$

44. The value of $\tan(1°) + \tan(89°)$ is equal to
(a) $\frac{1}{\sin 1°}$ (b) $\frac{2}{\sin 2°}$ (c) $\frac{2}{\sin 1°}$ (d) $\frac{1}{\sin 2°}$
(e) $\frac{\sin 2°}{2}$

45. Let $s_n = \cos\left(\frac{n\pi}{10}\right)$, $n = 1, 2, 3, \ldots$. Then the value of $\frac{s_1 s_2 \ldots\ldots s_{10}}{s_1 + s_2 + \ldots\ldots + s_{10}}$ is equal to
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $2\sqrt{2}$ (d) 0
(e) $\frac{1}{2}$

46. $\cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right) =$
(a) $\frac{3\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{-7\pi}{5}$ (d) $\frac{7\pi}{5}$
(e) $\frac{-2\pi}{5}$

47. The value of $\sec^2(\tan^{-1}3) + \text{cosec}^2(\cot^{-1}2)$ is equal to
(a) 5 (b) 13 (c) 15 (d) 23
(e) 25

48. If $\sin\theta + \text{cosec}\theta = 2$, then the value of $\sin^6\theta + \text{cosec}^6\theta$ is equal to
(a) 0 (b) 1 (c) 2 (d) 2^3
(e) 2^6

49. If $0 < x < \pi$, then
$$\frac{\sin 8x + 7\sin 6x + 18\sin 4x + 12\sin 2x}{\sin 7x + 6\sin 5x + 12\sin 3x} =$$
(a) $2\sin x$ (b) $\sin x$
(c) $\sin 2x$ (d) $2\cos x$
(e) $\cos x$

50. The points (2, 5) and (5, 1) are the two opposite vertices of a rectangle. If the other two vertices are points on the straight line $y = 2x + k$, then the value of k is
(a) 4 (b) 3 (c) –4 (d) –3
(e) 1

51. The circumcentre of the triangle with vertices (8, 6), (8, –2) and (2, –2) is at the point
(a) (2, –1) (b) (1, –2)
(c) (5, 2) (d) (2, 5)
(e) (4, 5)

52. The ratio by which the line $2x + 5y - 7 = 0$ divides the straight line joining the points (–4, 7) and (6, –5) is
(a) 1 : 4 (b) 1 : 2 (c) 1 : 1 (d) 2 : 3
(e) 1 : 3

53. The number of points (a, b), where a and b are positive integers, lying on the hyperbola $x^2 - y^2 = 512$ is
(a) 3 (b) 4 (c) 5 (d) 6
(e) 7

54. If p is the length of the perpendicular from the origin to the line whose intercepts with the

coordinate axes are $\frac{1}{3}$ and $\frac{1}{4}$, then the value of p is

(a) $\frac{3}{4}$ (b) $\frac{1}{12}$ (c) 5 (d) 12
(e) $\frac{1}{5}$

55. The slope of the straight line joining the centre of the circle $x^2 + y^2 - 8x + 2y = 0$ and the vertex of the parabola $y = x^2 - 4x + 10$ is

(a) $\frac{-5}{2}$ (b) $\frac{-7}{2}$ (c) $\frac{-3}{2}$ (d) $\frac{5}{2}$
(c) $\frac{7}{2}$

56. A straight line perpendicular to the line $2x + y = 3$ is passing through (1, 1). Its y-intercept is

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$
(e) $\frac{1}{3}$

57. If p and q are respectively the perpendiculars from the origin upon the straight lines whose equations are $x\sec\theta + y\operatorname{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$, then $4p^2 + q^2$ is equal to

(a) $5a^2$ (b) $4a^2$ (c) $3a^2$ (d) $2a^2$
(e) a^2

58. The shortest distance between the circles $(x - 1)^2 + (y + 2)^2 = 1$ and $(x + 2)^2 + (y - 2)^2 = 4$ is

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

59. The centre of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at (5, 5) is

(a) (10, 5) (b) (5, 8)
(c) (5, 10) (d) (8, 9)
(e) (9, 8)

60. A circle passes through the points (0, 0) and (0, 1) and also touches the circle $x^2 + y^2 = 16$. The radius of the circle is

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

61. A circle of radius $\sqrt{8}$ is passing through origin and the point (4, 0). If the centre lies on the line $y = x$, then the equation of the circle is

(a) $(x - 2)^2 + (y - 2)^2 = 8$
(b) $(x + 2)^2 + (y + 2)^2 = 8$
(c) $(x - 3)^2 + (y - 3)^2 = 8$
(d) $(x + 3)^2 + (y + 3)^2 = 8$
(e) $(x - 4)^2 + (y - 4)^2 = 8$

62. The parametric form of the ellipse $4(x + 1)^2 + (y - 1)^2 = 4$ is

(a) $x = \cos\theta - 1, y = 2\sin\theta - 1$
(b) $x = 2\cos\theta - 1, y = \sin\theta + 1$
(c) $x = \cos\theta - 1, y = 2\sin\theta + 1$
(d) $x = \cos\theta + 1, y = 2\sin\theta + 1$
(e) $x = \cos\theta + 1, y = 2\sin\theta - 1$

63. A point P on an ellipse is at a distance 6 units from a focus. If the eccentricity of the ellipse is $\frac{3}{5}$, then the distance of P from the corresponding directrix is

(a) $\frac{8}{5}$ (b) $\frac{5}{8}$ (c) 10 (d) 12
(e) 15

64. If the length of the latus rectum and the length of transverse axis of a hyperbola are $4\sqrt{3}$ and $2\sqrt{3}$ respectively, then the equation of the hyperbola is

(a) $\frac{x^2}{3} - \frac{y^2}{4} = 1$ (b) $\frac{x^2}{3} - \frac{y^2}{9} = 1$
(c) $\frac{x^2}{6} - \frac{y^2}{9} = 1$ (d) $\frac{x^2}{6} - \frac{y^2}{3} = 1$
(e) $\frac{x^2}{3} - \frac{y^2}{6} = 1$

65. If the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{5}{4}$ and $2x + 3y - 6 = 0$ is a focal chord of the hyperbola, then the length of transverse axis is equal to

(a) $\frac{12}{5}$ (b) 6 (c) $\frac{24}{7}$ (d) $\frac{24}{5}$
(e) $\frac{12}{7}$

66. The length of the transverse axis of a hyperbola is $2\cos\alpha$. The foci of the hyperbola are the same as that of the ellipse $9x^2 + 16y^2 = 144$. The equation of the hyperbola is

(a) $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{7-\cos^2\alpha} = 1$

(b) $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{7+\cos^2\alpha} = 1$

(c) $\frac{x^2}{1+\cos^2\alpha} - \frac{y^2}{7-\cos^2\alpha} = 1$

(d) $\frac{x^2}{1+\cos^2\alpha} - \frac{y^2}{7+\cos^2\alpha} = 1$

(e) $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{5-\cos^2\alpha} = 1$

67. If $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}, |\vec{b}| = 5$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then the area of the triangle formed by these two vectors as two sides is

(a) $\frac{15}{4}$ (b) $\frac{15}{2}$ (c) 15 (d) $\frac{15\sqrt{3}}{2}$

(e) $15\sqrt{3}$

68. If $\vec{a}\cdot\vec{b} = 0$ and $\vec{a} + \vec{b}$ makes an angle of 60° with $\vec{a}$, then

(a) $|\vec{a}| = 2|\vec{b}|$ (b) $2|\vec{a}| = |\vec{b}|$

(c) $|\vec{a}| = \sqrt{3}|\vec{b}|$ (d) $|\vec{a}| = |\vec{b}|$

(e) $\sqrt{3}|\vec{a}| = |\vec{b}|$

69. If $\hat{i} + \hat{j}, \hat{j} + \hat{k}, \hat{i} + \hat{k}$ are the position vectors of the vertices of a triangle ABC taken in order, then $\angle A$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

(e) $\frac{\pi}{3}$

70. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a}\cdot\vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$, then $|\vec{b}| =$

(a) $\sqrt{7}$ (b) $\sqrt{3}$ (c) 7 (d) 3

(e) $7\sqrt{3}$

71. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero vectors such that each one of them being perpendicular to the sum of the other two vectors, then the value of $|\vec{a} + \vec{b} + \vec{c}|^2$ is

(a) $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

(b) $|\vec{a}| + |\vec{b}| + |\vec{c}|$

(c) $2\left(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right)$

(d) $\frac{1}{2}\left(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right)$

(e) 0

72. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{0}$. If $|\vec{u}| = 3, |\vec{v}| = 4$ and $|\vec{w}| = 5$, then

$\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{u} =$

(a) 0 (b) –25 (c) 25 (d) 50

(e) 47

73. If $\lambda(3\hat{i} + 2\hat{j} - 6\hat{k})$ is a unit vector, then the values of λ are

(a) $\pm\frac{1}{7}$ (b) ± 7 (c) $\pm\sqrt{43}$ (d) $\pm\frac{1}{\sqrt{43}}$

(e) $\pm\frac{1}{\sqrt{7}}$

74. If the direction cosines of a vector of magnitude 3 are $\frac{2}{3}, \frac{-a}{3}, \frac{2}{3}, a > 0$, then the vector is

(a) $2\hat{i} + \hat{j} + 2\hat{k}$ (b) $2\hat{i} - \hat{j} + 2\hat{k}$

(c) $\hat{i} - 2\hat{j} + 2\hat{k}$ (d) $\hat{i} + 2\hat{j} + 2\hat{k}$

(e) $\hat{i} + 2\hat{j} - 2\hat{k}$

75. Equation of the plane through the mid-point of the line segment joining the points $P(4, 5, -10)$ and $Q(-1, 2, 1)$ and perpendicular to PQ is

(a) $\vec{r}\cdot\left(\frac{3}{2}\hat{i} + \frac{7}{2}\hat{j} - \frac{9}{2}\hat{k}\right) = 45$

(b) $\vec{r}\cdot(-\hat{i} + 2\hat{j} + \hat{k}) = \frac{135}{2}$

(c) $\vec{r}\cdot(5\hat{i} + 3\hat{j} - 11\hat{k}) + \frac{135}{2} = 0$

(d) $\vec{r}\cdot(4\hat{i} + 5\hat{j} - 10\hat{k}) = 85$

(e) $\vec{r}\cdot(5\hat{i} + 3\hat{j} - 11\hat{k}) = \frac{135}{2}$

76. The angle between the straight lines $x-1=\frac{2y+3}{3}=\frac{z+5}{2}$ and $x=3r+2; y=-2r-1;$ $z=2$, where r is a parameter, is

(a) $\frac{\pi}{4}$ (b) $\cos^{-1}\left(\frac{-3}{\sqrt{182}}\right)$

(c) $\sin^{-1}\left(\frac{-3}{\sqrt{182}}\right)$ (d) $\frac{\pi}{2}$

(e) 0

77. Equation of the line through the point (2, 3, 1) and parallel to the line of intersection of the planes $x - 2y - z + 5 = 0$ and $x + y + 3z = 6$ is

(a) $\frac{x-2}{-5}=\frac{y-3}{-4}=\frac{z-1}{3}$

(b) $\frac{x-2}{5}=\frac{y-3}{-4}=\frac{z-1}{3}$

(c) $\frac{x-2}{5}=\frac{y-3}{4}=\frac{z-1}{3}$

(d) $\frac{x-2}{4}=\frac{y-3}{3}=\frac{z-1}{2}$

(e) $\frac{x-2}{-4}=\frac{y-3}{-3}=\frac{z-1}{2}$

78. A unit vector parallel to the straight line $\frac{x-2}{3}=\frac{3+y}{-1}=\frac{z-2}{-4}$ is

(a) $\frac{1}{\sqrt{26}}(3\hat{i}-\hat{j}+4\hat{k})$ (b) $\frac{1}{\sqrt{26}}(\hat{i}+3\hat{j}-\hat{k})$

(c) $\frac{1}{\sqrt{26}}(3\hat{i}-\hat{j}-4\hat{k})$ (d) $\frac{1}{\sqrt{26}}(3\hat{i}+\hat{j}+4\hat{k})$

(e) $\frac{1}{\sqrt{26}}(\hat{i}-3\hat{j}+4\hat{k})$

79. The angle between a normal to the plane $2x - y + 2z - 1 = 0$ and the z-axis is

(a) $\cos^{-1}\left(\frac{1}{3}\right)$ (b) $\sin^{-1}\left(\frac{2}{3}\right)$

(c) $\cos^{-1}\left(\frac{2}{3}\right)$ (d) $\sin^{-1}\left(\frac{1}{3}\right)$

(e) $\sin^{-1}\left(\frac{3}{5}\right)$

80. Foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z = 29$ is

(a) (5, –1, 4) (b) (7, –1, 3)

(c) (5, –2, 3) (d) (2, –3, 4)

(e) (1, –3, 4)

81. The distance between the x-axis and the point (3, 12, 5) is

(a) 3 (b) 13 (c) 14 (D) 12

(e) 5

82. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is

(a) 9 (b) 4 (c) 3 (d) 2

(e) 1

83. If two dice are thrown simultaneously, then the probability that the sum of the numbers which come up on the dice to be more than 5 is

(a) $\frac{5}{36}$ (b) $\frac{1}{6}$ (c) $\frac{5}{18}$ (d) $\frac{7}{18}$

(e) $\frac{13}{18}$

84. Let A and B be two events such that $P(A \cup B) = P(A) + P(B) - P(A)P(B)$. If $0 < P(A) < 1$ and $0 < P(B) < 1$, then $P(A \cup B)' =$

(a) $1 - P(A)$ (b) $1 - P(A')$

(c) $1 - P(A)P(B)$ (d) $[1 - P(A)]P(B')$

(e) 1

85. The standard deviation of 9, 16, 23, 30, 37, 44, 51 is

(a) 7 (b) 9 (c) 12 (d) 14

(e) 16

86. The value of $\lim_{x\to 3}\frac{x^5-3^5}{x^8-3^8}$ is equal to

(a) $\frac{5}{8}$ (b) $\frac{5}{64}$ (c) $\frac{5}{216}$ (d) $\frac{1}{27}$

(e) $\frac{1}{63}$

87. Let $f(x) = (x^5 - 1)(x^3 + 1), g(x) = (x^2 - 1)(x^2 - x + 1)$ and let $h(x)$ be such that $f(x) = g(x)h(x)$. Then $\lim_{x\to 1} h(x)$ is

(a) 0 (b) 1 (c) 3 (d) 4

(e) 5

88. $\lim_{x\to 0} \frac{\log(1+3x^2)}{x(e^{5x}-1)} =$

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{-3}{5}$ (d) $\frac{-5}{3}$

(e) 1

89. If $f(x)=\frac{x+2}{3x-1}$, then $f(f(x))$ is

(a) x (b) $-x$ (c) $\frac{1}{x}$ (d) $-\frac{1}{x}$

(e) 0

90. Let $f(x)=\begin{cases} ax+3, & x\le 2 \\ a^2x-1, & x>2 \end{cases}$. Then the values of a for which f is continuous for all x are

(a) 1 and –2 (b) 1 and 2

(c) –1 and 2 (d) –1 and –2

(e) 0 and 3

91. Let R be the set of all real numbers. Let $f: R \to R$ be a function such that $|f(x)-f(y)|^2 \le |x-y|^3,\ \forall\ x, y \in R$. Then $f'(x) =$

(a) $f(x)$ (b) 1 (c) 0 (d) x^2

(e) x

92. Let $f(x)=\int_1^x \sin^2\left(\frac{t}{2}\right)dt$. Then the value of $\lim_{x\to 0}\frac{f(\pi+x)-f(\pi)}{x}$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

(e) 0

93. If $y = f(x^2 + 2)$ and $f'(3) = 5$, then $\frac{dy}{dx}$ at $x = 1$ is

(a) 5 (b) 25 (c) 15 (d) 20

(e) 10

94. Let $f(x) = x^2 + bx + 7$. If $f'(5)=2f'\left(\frac{7}{2}\right)$, then the value of b is

(a) 4 (b) 3 (c) –4 (d) –3

(e) 2

95. If $x = \sin t$ and $y = \tan t$, then $\frac{dy}{dx} =$

(a) $\cos^3 t$ (b) $\frac{1}{\cos^3 t}$

(c) $\frac{1}{\cos^2 t}$ (d) $\sin^2 t$

(e) $\frac{1}{\sin^2 t}$

96. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then $1+\left(\frac{dy}{dx}\right)^2$ is

(a) $\tan\theta$ (b) $\tan^2\theta$

(c) 1 (d) $\sec^2\theta$

(e) $\sec\theta$

97. If $y=\sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{2}{\sqrt{1-x^2}}$ (d) $\frac{2x}{\sqrt{1-x^2}}$

(e) $\frac{-2x}{\sqrt{1-x^2}}$

98. A straight line parallel to the line $2x - y + 5 = 0$ is also a tangent to the curve $y^2 = 4x + 5$. Then the point of contact is

(a) (2, 1) (b) (–1, 1)

(c) (1, 3) (d) (3, 4)

(e) (–1, 2)

99. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is strictly decreasing in the interval

(a) (2, 3) (b) $(-\infty, 2)$

(c) (3, 4) (d) $(-\infty, 3) \cup (4, \infty)$

(e) $(-\infty, 2) \cup (3, \infty)$

100. The slope of the tangent to the curve $y^2e^{xy} = 9e^{-3}x^2$ at (–1, 3) is

(a) $\frac{-15}{2}$ (b) $\frac{-9}{2}$ (c) 15 (d) $\frac{15}{2}$

(e) $\frac{9}{2}$

101. The radius of a cylinder is increasing at the rate of 5 cm/min so that its volume is constant. When its radius is 5 cm and height is 3 cm the rate of decreasing of its height is

(a) 6 cm/min (b) 3 cm/min

(c) 4 cm/min (d) 5 cm/min

(e) 2 cm/min

102. The function $f(x)=\begin{cases}2x^2-1, & \text{if } 1\le x\le 4\\ 151-30x, & \text{if } 4<x\le 5\end{cases}$ is not suitable to apply Rolle's theorem since

(a) $f(x)$ is not continuous on $[1, 5]$
(b) $f(1)\neq f(5)$
(c) $f(x)$ is continuous only at $x=4$
(d) $f(x)$ is not differentiable in $(4, 5)$
(e) $f(x)$ is not differentiable at $x=4$

103. The slope of the normal to the curve $y=x^2-\frac{1}{x^2}$ at $(-1, 0)$ is

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) 4 (d) -4
(e) 0

104. The minimum value of $\sin x+\cos x$ is

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}}$
(e) 1

105. $\int \frac{1}{x^2(x^4+1)^{3/4}}\,dx$ is equal to

(a) $-\frac{(1+x^4)^{3/4}}{x}+C$ (b) $-\frac{(1+x^4)^{1/4}}{2x}+C$
(c) $-\frac{(1+x^4)^{1/4}}{x}+C$ (d) $-\frac{(1+x^4)^{1/4}}{x^2}+C$
(e) $-\frac{(1+x^4)^{1/2}}{x}+C$

106. $\int \frac{(1+x)e^x}{\sin^2(xe^x)}\,dx$ is equal to

(a) $-\cot(e^x)+C$ (b) $\tan(xe^x)+C$
(c) $\tan(e^x)+C$ (d) $\cot(xe^x)+C$
(e) $-\cot(xe^x)+C$

107. $\int \frac{xe^x}{(1+x)^2}\,dx$ is equal to

(a) $\frac{-e^x}{x+1}+C$ (b) $\frac{e^x}{x+1}+C$
(c) $\frac{xe^x}{x+1}+C$ (d) $\frac{-xe^x}{x+1}+C$
(e) $\frac{e^x}{(x+1)^2}+C$

108. $\int e^x(\sin x+2\cos x)\sin x\,dx$ is equal to

(a) $e^x\cos x+C$ (b) $e^x\sin x+C$
(c) $e^x\sin^2 x+C$ (d) $e^x\sin 2x+C$
(e) $e^x(\cos x+\sin x)+C$

109. $\int \sqrt{1+\cos x}\,dx$ is equal to

(a) $2\sin\left(\frac{x}{2}\right)+C$ (b) $\sqrt{2}\sin\left(\frac{x}{2}\right)+C$
(c) $\frac{1}{2}\sin\left(\frac{x}{2}\right)+C$ (d) $\frac{\sqrt{2}}{2}\sin\left(\frac{x}{2}\right)+C$
(e) $2\sqrt{2}\sin\left(\frac{x}{2}\right)+C$

110. $\int \frac{\sqrt{x^2-1}}{x}\,dx$ is equal to

(a) $\sqrt{x^2-1}-\sec^{-1}x+C$
(b) $\sqrt{x^2-1}+\tan^{-1}x+C$
(c) $\sqrt{x^2-1}+\sec^{-1}x+C$
(d) $\sqrt{x^2-1}-\tan x+C$
(e) $\sqrt{x^2-1}+\sec x+C$

111. $\int \frac{\sqrt{5+x^2}}{x^4}\,dx$ is equal to

(a) $\frac{1}{15}\left(1+\frac{5}{x^2}\right)^{3/2}+C$
(b) $\frac{-1}{15}\left(1+\frac{1}{x^2}\right)^{3/2}+C$
(c) $\frac{-1}{15}\left(1+\frac{5}{x^2}\right)^{3/2}+C$
(d) $\frac{1}{15}\left(1+\frac{1}{x^2}\right)^{3/2}+C$
(e) $\frac{-1}{10}\left(1+\frac{1}{x^2}\right)^{3/2}+C$

112. The value of $\int_0^1 \frac{dx}{e^x+e}$ is equal to

(a) $\frac{1}{e}\log\left(\frac{1+e}{2}\right)$ (b) $\log\left(\frac{1+e}{2}\right)$

(c) $\frac{1}{e}\log(1+e)$ (d) $\log\left(\frac{2}{1+e}\right)$

(e) $\frac{1}{e}\log\left(\frac{2}{1+e}\right)$

113. Area bounded by the curves $y = e^x$, $y = e^{-x}$ and the straight line $x = 1$ is (in sq. units)

(a) $e+\frac{1}{e}$ (b) $e+\frac{1}{e}+2$

(c) $e+\frac{1}{e}-2$ (d) $e-\frac{1}{e}+2$

(e) $e-\frac{1}{e}$

114. The value of the integral $\int_1^e \frac{1+\log x}{3x}\,dx$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) e

(e) $\frac{1}{e}$

115. The value of the integral $\int_0^1 \frac{x^3}{1+x^8}\,dx$ is equal to

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{16}$ (d) $\frac{\pi}{6}$

(e) $\frac{\pi}{12}$

116. The value of the integral $\int_2^4 \left(\frac{\log t}{t}\right)dt$ is equal to

(a) $\frac{1}{2}(\log 2)^2$ (b) $\frac{5}{2}(\log 2)^2$

(c) $\frac{3}{2}(\log 2)^2$ (d) $(\log 2)^2$

(e) $\frac{3}{2}(\log 2)$

117. The solution of the differential equation $(kx - y^2)dy = (x^2 - ky)dx$ is

(a) $x^3 - y^3 = 3kxy + C$

(b) $x^3 + y^3 = 3kxy + C$

(c) $x^2 - y^2 = 2kxy + C$

(d) $x^2 + y^2 = 2kxy + C$

(e) $x^3 - y^2 = 3kxy + C$

118. The solution of the differential equation $\frac{dy}{dx} = e^x + 1$ is

(a) $y = e^x + C$ (b) $y = x + e^x + C$

(c) $y = xe^x + C$ (d) $y = x(e^x + 1) + C$

(e) $y = e^x + Cx$

119. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{3/2} = y$ are respectively

(a) 1, 1 (b) 1, 2 (c) 1, 3 (d) 2, 1

(e) 2, 2

120. An integrating factor of the differential equation $\sin x\frac{dy}{dx} + 2y\cos x = 1$ is

(a) $\sin^2 x$ (b) $\frac{2}{\sin x}$

(c) $\log|\sin x|$ (d) $\frac{1}{\sin^2 x}$

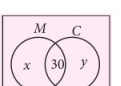
(e) $2\sin x$

SOLUTIONS

1. (b) : $2 \oplus 3 = 2^2 + 3^2 = 13$

Now, $13 \oplus 4 = 169 + 16 = 185$

2. (a) : Let x be the number of students who take mathematics and y be the number of students who take chemistry.



$30 = \frac{10}{100}(x + 30) \Rightarrow x = 270$

Similarly, $30 = \frac{12}{100}(y + 30) \Rightarrow y = 220$

Number of students who take atleast one of these two subjects = 270 + 220 + 30 = 520

3. (d) : $f(x) = |x - 2|$

$f(1) = 1, f(3) = 1$

So, $f(x)$ is not 1 – 1 function.

4. (e): Number of one-one functions from A to B

$= {}^nC_m \times m!$ where $n \geq m$

$= {}^8C_4 \times 4! = 1680$

5. (None of the options is correct)

We have $f(x) = x^2 + 2x + 2$

$= (x^2 + 2x + 2) \geq 1 \ \forall \ x \in R$

$\Rightarrow \ 1 \leq f(x) < \infty \ \forall \ x \in R$

Hence, range $(f) = [1, \infty)$

6. (None of the options is correct)

$f(x) = \sqrt{x}$ and $g(x) = 2x - 3$

$(fog)(x) = f(g(x)) = f(2x - 3) = \sqrt{2x - 3}$

Note : If domain of $fog(x)$ would have been asked then option (e) would have been correct.

7. (b) : We have, $z = \dfrac{(\sqrt{3} + i)^3 \, (3i + 4)^2}{(8 + 6i)^2}$

$$= \frac{[3\sqrt{3} + i^3 + 3\sqrt{3}i\,(\sqrt{3} + i)][9i^2 + 16 + 24i]}{(64 + 36i^2 + 96i]}$$

$$= \frac{(3\sqrt{3} - i + 9i - 3\sqrt{3})\,(7 + 24i)}{28 + 96i}$$

$$= \frac{(8i)\,(7 + 24i)}{28 + 96i} = \frac{56i - 192}{28 + 96i} = \frac{(56i - 192)\,(28 - 96i)}{(28)^2 - (96)^2}$$

$$= \frac{1568i + 5376 - 5376 + 18432i}{784 + 9216} = 2i$$

$|z| = \sqrt{0^2 + 2^2} = 2$

8. (d) : $w = u + iv$

$$z = \frac{w - 1}{w + 1} = \frac{u + iv - 1}{u + iv + 1}$$

$$= \frac{(u - 1) + iv}{(u + 1) + iv} = \frac{[(u - 1) + iv][(u + 1) - iv]}{[(u + 1) + iv][(u + 1) - iv]}$$

$$= \frac{u^2 - 1 - iv\,(u - 1) + iv\,(u + 1) + v^2}{(u + 1)^2 - i^2v^2}$$

$$= \frac{u^2 + v^2 - 1 + iv + iv}{u^2 + 1 + 2u + v^2}$$

$$= \frac{2iv}{2 + 2u} = \frac{iv}{1 + u} \quad \left(\because \ |w| = \sqrt{u^2 + v^2} = 1\right)$$

So, $\mathrm{Re}(z) = 0$

9. (a) : $e^{2\pi i/3} = \cos\dfrac{2\pi}{3} + \dfrac{i \sin 2\pi}{3} = \omega$

Now, $1 + z + 3z^2 + 2z^3 + 2z^4 + 3z^5$

$= 1 + \omega + 3\omega^2 + 2\omega^3 + 2\omega^4 + 3\omega^5$

$= 1 + \omega + 3\omega^2 + 2 + 2\omega + 3\omega^2$

$= 3 + 3\omega + 6\omega^2 = 3(1 + \omega + \omega^2) + 3\omega^2$

$= 3\omega^2 = 3e^{4\pi i/3}$

$$= 3\left(\cos\frac{4\pi}{3} + i \sin\frac{4\pi}{3}\right)$$

$$= 3\left[\cos\left(\pi + \frac{\pi}{3}\right) + i \sin\left(\pi + \frac{\pi}{3}\right)\right]$$

$$= 3\left[-\cos\frac{\pi}{3} - i \sin\frac{\pi}{3}\right] = -3e^{i\pi/3}$$

10. (d) : We have $z_1 = 2\sqrt{2}(1 + i)$

$z_1^2 = 8(1 + i)^2 = 8(1 + i^2 + 2i) = 16i$

Also, $z_2 = 1 + i\sqrt{3}$

$\Rightarrow \ z_2^3 = (1 + i\sqrt{3})^3 = 1 + 3\sqrt{3}i^3 + 3\sqrt{3}i(1 + i\sqrt{3})$

$= 1 - 3\sqrt{3}i + 3\sqrt{3}i + 9i^2 = -8$

So, $z_1^2 z_2^3 = 16 \cdot (-8)i = -128i$

11. (a) : $(z_3 - z_1) = (z_2 - z_1)\,[\cos 90° + i \sin 90°]$

$(z_3 - z_1) = i(z_2 - z_1)$

$(z_3 - z_1)^2 = -(z_2 - z_1)^2$

$\Rightarrow \ (z_3 - z_1)^2 + (z_2 - z_1)^2 = 0$

12. (b) : Let α, $\alpha + 2$ be two consecutive odd roots

$\therefore \ \alpha + \alpha + 2 = a$ and $\alpha(\alpha + 2) = b$

$\therefore \ 2\alpha + 2 = a$ ....(1) and $\alpha^2 + 2\alpha = b$ ...(2)

Now, $\alpha^2 + 2\alpha = b$

$$\Rightarrow \ \left(\frac{a - 2}{2}\right)^2 + 2\left(\frac{a - 2}{2}\right) = b \ \text{(Using (1))}$$

$$\Rightarrow \ \frac{a^2 + 4 - 4a}{4} + a - 2 = b$$

$\Rightarrow \ a^2 - 4a + 4 + 4a - 8 = 4b$

$\Rightarrow \ a^2 - 4 = 4b \ \Rightarrow \ a^2 - 4b = 4$

13. (b) : $\alpha + \beta = a$ and $\alpha\beta = b^2$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow \ \alpha^2 + \beta^2 = a^2 - 2b^2$

14. (b) : $\alpha + \beta = -3$ and $\alpha\beta = -4$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{3}{4}$$

15. (b) : We have $3^{2x} - 18 \cdot 3^x + 81 = 0$ .....(1)

Let $3^x = y$, so equation (1) becomes

$y^2 - 18y + 81 = 0$

$\Rightarrow \ (y - 9)^2 = 0 \ \Rightarrow \ y = 9$

Now, $3^x = 3^2 \ \Rightarrow \ x = 2$

16. (a) : $\alpha+\beta=-2b$ and $\alpha\beta=c$

So, $b^2-c=\left(\frac{\alpha+\beta}{2}\right)^2-\alpha\beta=\frac{(\alpha+\beta)^2-4\alpha\beta}{4}$

$=\frac{\alpha^2+\beta^2+2\alpha\beta-4\alpha\beta}{4}=\frac{(\alpha-\beta)^2}{4}$

17. (d) : $\alpha+\beta=-\frac{3}{2}$ and $\alpha\beta=\frac{1}{2}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{9}{4}-1=\frac{5}{4}$

Also, $\alpha^2\beta^2=\frac{1}{4}$

So, required equation is $x^2-\frac{5}{4}x+\frac{1}{4}=0$

$\Rightarrow\ 4x^2-5x+1=0$

18. (d) : $\sum_{n=8}^{17}\frac{1}{(n+2)(n+3)}=\sum_{n=8}^{17}\frac{(n+3)-(n+2)}{(n+2)(n+3)}$

$=\sum_{n=8}^{17}\frac{1}{n+2}-\sum_{n=8}^{17}\frac{1}{n+3}$

$=\left[\frac{1}{10}-\frac{1}{11}\right]+\left[\frac{1}{11}-\frac{1}{12}\right]+.....+\left[\frac{1}{19}-\frac{1}{20}\right]$

$=\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$

19. (d) : We have $a:b=3+2\sqrt{2}:3-2\sqrt{2}$

$\frac{a+b}{2}=3$ and $\sqrt{ab}=1$

So, required ratio = 3 : 1

20. (a) : We have $x_1+x_4+x_9+x_{11}+x_{20}+x_{22}+x_{27}+x_{30}=272$

$\Rightarrow\ a+a+3d+a+8d+a+10d+a+19d+a+21d+a+26d+a+29d=272$

$\Rightarrow\ 8a+116d=272\ \Rightarrow\ 2a+29d=68$

Also, $S_{30}=\frac{30}{2}[2a+29d]=15\times68=1020$

21. (d) : We have, $ar=24$ and $ar^4=3$

So, $r^3=\frac{3}{24}\ \Rightarrow\ r=\frac{1}{2}$ and $a=48$

Now, $a+ar+ar^2+ar^3+ar^4+ar^5$

$=a(1+r+r^2+r^3+r^4+r^5)$

$=48\left(1+\frac{1}{2}+\frac{1}{2^2}+.....+\frac{1}{2^5}\right)$

$=48\frac{\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}}=48\left(\frac{2^6-1}{2^6}\right)\cdot2=\frac{189}{2}$

22. (e) : Now, $S_{75}=\frac{75}{2}\left[2a+(n-1)d\right]$

$2625=\frac{75}{2}[2a+(74)d]$

$\frac{2625\times2}{75\times2}=(a+37d)\ \Rightarrow\ a+37d=35$

23. (b) : $2k=-5-1$ ($\because$ terms are in A.P.)

$\Rightarrow\ 2k=-6\ \Rightarrow\ k=-3$

24. (e) : Here, $T_n={}^nC_3$

Now, $T_{n+1}-T_n={}^{n+1}C_3-{}^nC_3$

$\Rightarrow\ 36={}^nC_2$

$\Rightarrow\ \frac{n(n-1)}{2}=36\ \Rightarrow\ n(n-1)=72$

$\Rightarrow\ n^2-n-72=0\ \Rightarrow\ (n-9)(n+8)=0$

$\Rightarrow\ n=9\qquad(n\neq-8)$

25. (a) : Here $n=10$, which is an even number.

$\left[\frac{10}{2}+1\right]^{\text{th}}$ term *i.e.* 6th term is the middle term

Hence, the middle term $=T_6=T_{5+1}$

$={}^{10}C_5\left(\frac{10}{x}\right)^5\left(\frac{x}{10}\right)^5={}^{10}C_5$

26. (b) : $x^{49}(-1-2-3\ .....\ -50)$

$=-x^{49}(1+2+3+.....+50)$

$=-x^{49}\left(\frac{50\times51}{2}\right)=-1275x^{49}$

27. (b) :

$\left(\frac{1}{x}+2x\right)^6={}^6C_0\left(\frac{1}{x}\right)^6(2x)^0+{}^6C_1\left(\frac{1}{x}\right)^5(2x)^1$

$+{}^6C_2\left(\frac{1}{x}\right)^4(2x)^2+{}^6C_3\left(\frac{1}{x}\right)^3(2x)^3$

$+{}^6C_4\left(\frac{1}{x}\right)^2(2x)^4+{}^6C_5\left(\frac{1}{x}\right)(2x)^5$

$+{}^6C_6\left(\frac{1}{x}\right)^0(2x)^6$

$$=\left(\frac{1}{x}\right)^6+12\left(\frac{1}{x}\right)^5 x+15\times 4\left(\frac{1}{x^4}\right)(x^2)$$
$$+20\times 8\left(\frac{1}{x}\right)^3(x^3)+15\times 16\left(\frac{1}{x^2}\right)x^4$$
$$+6\left(\frac{1}{x}\right)\times 32x^5+64x^6$$

$$=\left(\frac{1}{x}\right)^6+12\left(\frac{1}{x}\right)^4+60\left(\frac{1}{x}\right)^2+160+240x^2$$
$$+192x^4+64x^6$$

Sum of coefficients = 1 + 12 + 60 + 160 + 240
$$+192+64=729$$

28. (e) : Now, ${}^2P_1+{}^3P_1+\ldots\ldots\ldots+{}^nP_1$
$=2+3+\ldots\ldots\ldots+n$
$$=\frac{n(n+1)}{2}-1=\frac{n^2+n-2}{2}$$

29. (c): a is odd means $a = 1, 3, 5, 7, 9$
b is divisible by 3 means $b = 0, 3, 6, 9$
c is even means $c = 0, 2, 4, 6, 8$
d is prime means $d = 2, 3, 5, 7$
Total number of four digit numbers
$= {}^5C_1\times{}^4C_1\times{}^5C_1\times{}^4C_1$
$= 5\times 4\times 5\times 4 = 400$

30. (e) : Applying $R_2\to R_2-R_1$ and $R_3\to R_3-R_2$, we get

$$\begin{vmatrix} a_1 & a_2 & 1\\ a_2-a_1 & a_3-a_2 & 0\\ a_3-a_2 & a_4-a_3 & 0\end{vmatrix}$$

$$=\begin{vmatrix} a_1 & a_2 & 1\\ d & d & 0\\ d & d & 0\end{vmatrix}=0 \quad (\because\ d=a_2-a_1=a_3-a_2\ldots..)$$

31. (d) : Area of triangle $=\frac{1}{2}\begin{vmatrix} \frac{x_1}{a} & \frac{y_1}{a} & 1\\ \frac{x_2}{b} & \frac{y_2}{b} & 1\\ \frac{x_3}{c} & \frac{y_3}{c} & 1\end{vmatrix}$

$$=\frac{1}{2abc}\begin{vmatrix} x_1 & y_1 & a\\ x_2 & y_2 & b\\ x_3 & y_3 & c\end{vmatrix}$$

$$=\frac{1}{4abc}\begin{vmatrix} x_1 & y_1 & 2a\\ x_2 & y_2 & 2b\\ x_3 & y_3 & 2c\end{vmatrix}=\frac{1}{4abc}\times\frac{abc}{2}=\frac{1}{8}$$

32. (d) : $\begin{vmatrix} 3 & 1 & -1\\ 1 & 0 & -1\\ 2 & 2 & a\end{vmatrix}\neq 0$

$\Rightarrow\ 3(2)-1(a+2)-1(2)\neq 0$
$\Rightarrow\ 6-a-2-2\neq 0\ \Rightarrow\ a\neq 2$

33. (e) : Since matrix is singular, so $|A| = 0$

$$\Rightarrow\ \begin{vmatrix} 2-k & 2\\ 1 & 3-k\end{vmatrix}=0$$

$(2-k)(3-k)-2=0$
$\Rightarrow\ 6-2k-3k+k^2-2=0$
$\Rightarrow\ k^2-5k+4=0\ \Rightarrow\ 5k-k^2=4$

34. (a) :
$$\begin{vmatrix} \log_a 1 & \log_a b & \log_a c\\ \log_a\left(\frac{1}{b}\right) & \log_b 1 & \log_a\frac{1}{c}\\ \log_a\left(\frac{1}{c}\right) & \log_a c & \log_c 1\end{vmatrix}$$

$$=\begin{vmatrix} \log 1 & \log b & \log c\\ -\log b & \log 1 & -\log c\\ -\log c & \log c & \log 1\end{vmatrix}$$

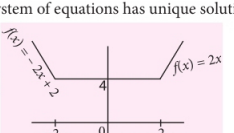
$$=\begin{vmatrix} 0 & \log b & \log c\\ -\log b & 0 & -\log c\\ -\log c & \log c & 0\end{vmatrix}$$

$= 0$

35. (a) :
$$\begin{vmatrix} 2 & 1 & -4\\ 3 & 2 & -2\\ 1 & 1 & 2\end{vmatrix}$$

$= 2(4+2)-1(6+2)-4(3-2)$
$= 12-8-4=0$
So, the system of equations has unique solution.

36. (d) :



37. (e)

38. (c): $\sim [p \vee (\sim q)] = \sim p \wedge (\sim (\sim q)) = (\sim p) \wedge q$

39. (e)

40. (d) : $p \vee (\sim q)$

41. (e) : $a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

$\Rightarrow \quad \sin\theta + \cos\theta \le \sqrt{1^2+1^2} \le \sqrt{2}$

42. (c): Now, $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) + \sin^{-1}\left(\frac{1}{3}\right)$

$= \tan^{-1}(2\sqrt{2}) + \tan^{-1}\left(\frac{1}{2\sqrt{2}}\right)$

$= \tan^{-1}(2\sqrt{2}) + \cot^{-1}(2\sqrt{2}) = \pi/2$

43. (e) : We have

$$\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) + \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = 2\tan^{-1}x$$

$\Rightarrow \quad 2\tan^{-1}a + 2\tan^{-1}b = 2\tan^{-1}x$

$$\left(\because \quad 2\tan^{-1}a = \cos^{-1}\left(\frac{1-a^2}{1+b^2}\right) \text{ and so on}\right)$$

$\Rightarrow \quad \tan^{-1}a + \tan^{-1}b = \tan^{-1}x$

$$\Rightarrow \quad \tan^{-1}\left(\frac{a+b}{1-ab}\right) = \tan^{-1}x \quad \Rightarrow \quad \frac{a+b}{1-ab} = x$$

44. (b) : $\tan(1°) + \tan(89°) = \tan 1° + \cot 1°$

$$= \frac{\sin^2 1° + \cos^2 1°}{\sin 1° \cos 1°} = \frac{2}{2\sin 1° \cos 1°} = \frac{2}{\sin 2°}$$

45. (d) : $s_5 = \cos\left(\frac{5\pi}{10}\right) = \cos\frac{\pi}{2} = 0$

46. (a): $\cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{3\pi}{5}\right)\right)$

$$= \cos^{-1}\left(\cos\left(\frac{3\pi}{5}\right)\right) = \frac{3\pi}{5}$$

47. (c): We have $\sec^2(\tan^{-1}3) + \text{cosec}^2(\cot^{-1}2)$

$= (\sec(\tan^{-1}3))^2 + (\text{cosec}(\cot^{-1}2))^2$

$= (\sec(\sec^{-1}\sqrt{10}))^2 + (\text{cosec}(\text{cosec}^{-1}\sqrt{5}))^2$

$= (\sqrt{10})^2 + (\sqrt{5})^2 = 15$

48. (c): We have $\sin\theta + \text{cosec}\theta = 2$

$\Rightarrow \quad \sin^2\theta + 1 = 2\sin\theta$

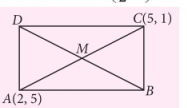
$\Rightarrow \quad (\sin\theta - 1)^2 = 0 \quad \Rightarrow \quad \sin\theta = 1$

So, $\sin^6\theta + \text{cosec}^6\theta = 1 + 1 = 2$

49. (d)

50. (c): Coordinates of $M = \left(\frac{5+2}{2}, \frac{1+5}{2}\right)$

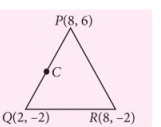
$$= \left(\frac{7}{2}, 3\right)$$



Since $y = 2x + k$ passes through M

So, $3 = 7 + k \quad \Rightarrow \quad k = -4$

51. (c): Let P, Q and R be the vertices of ΔPQR. Circumcentre of the triangle is the midpoint of hypotenuse.



Coordinates of C is $\left(\frac{8+2}{2}, \frac{6-2}{2}\right) = (5, 2)$

52. (c): The required ratio is

$$-\left[\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}\right] = -\left[\frac{2(-4) + 5(7) - 7}{2(6) + 5(-5) - 7}\right] = -[-1] = 1$$

53. (b) : $a^2 - b^2 = 512$

$\Rightarrow \quad (a+b)(a-b) = 2^9$

$\Rightarrow \quad (a+b, a-b) = (2^8, 2), (2^7, 2^2), (2^6, 2^3), (2^5, 2^4)$

$[(\because \ a > b, a + b > a - b)$, the other combination like $(2^4, 2^5)$ etc cannot be accepted.)

54. (e) : The line is $\frac{x}{(1/3)} + \frac{y}{(1/4)} = 1$

$\Rightarrow \quad 3x + 4y - 1 = 0$

So, $p = \left|\frac{3(0) + 4(0) - 1}{\sqrt{3^2 + 4^2}}\right| = \frac{1}{5}$

55. (b) : Centre of the circle $x^2 + y^2 - 8x + 2y = 0$ is $(4, -1)$

Given parabola is $y = (x-2)^2 + 6$

$\Rightarrow \quad (x-2)^2 = 4\cdot\frac{1}{4}(y-6)$

Vertex is (2, 6)

Required slope $= \left(\frac{6+1}{2-4}\right) = -\frac{7}{2}$

56. (d) : The required line perpendicular to the line $2x + y = 3$ is $x - 2y + \lambda = 0$

It passes through (1, 1). So,

$1 - 2 + \lambda = 0 \Rightarrow \lambda = 1$

So, required line is $x - 2y + 1 = 0$

Its y-intercept is $-\frac{c}{b} = \frac{1}{2}$

57. (e) : $p = \left|\frac{-a}{\sqrt{\sec^2\theta + \text{cosec}^2\theta}}\right|$

$= \frac{a}{\sqrt{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}}} = a\sin\theta\cos\theta = \frac{a}{2}\sin 2\theta$

$q = \left|-\frac{a\cos 2\theta}{\cos^2\theta + \sin^2\theta}\right| = a\cos 2\theta$

Now, $4p^2 + q^2 = a^2(\sin^2 2\theta + \cos^2 2\theta) = a^2$

58. (b) : Centre and radius of circle

$(x-1)^2 + (y+2)^2 = 1$ is $C_1(1, -2)$ and $r_1 = 1$

Centre and radius of circle $(x+2)^2 + (y-2)^2 = 4$ is $C_2(-2, 2)$ and $r_2 = 2$

$C_1C_2 = \sqrt{9+16} = 5 > r_1 + r_2$

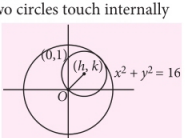
Two circles do not intersect

$d_{\min} = C_1C_2 - (r_1 + r_2) = 5 - (1 + 2) = 2$

59. (e) : $\left(\frac{h+1}{2}, \frac{k+2}{2}\right) = (5, 5)$

$\Rightarrow \quad h = 9, k = 8$

60. (b) : Two circles touch internally



$\Rightarrow \quad C_1C_2 = |r_1 - r_2|$

$\Rightarrow \quad \sqrt{h^2 + k^2} = \left|4 - \sqrt{h^2 + k^2}\right|$

$\Rightarrow \quad 2\sqrt{h^2 + k^2} = 4$

$\Rightarrow \quad \sqrt{h^2 + k^2} = 2 \Rightarrow r = 2$

61. (a) : Let the required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(1)

$\sqrt{g^2 + f^2 - c} = \sqrt{8} \Rightarrow g^2 + f^2 = 8$ ...(2)

$16 + 8g = 0 \Rightarrow g = -2$

Putting the value of g in (2), we get

$f^2 = 4 \Rightarrow f = -2$

Required circle is $x^2 + y^2 - 4x - 4y = 0$

$\Rightarrow (x-2)^2 + (y-2)^2 = 8$

62. (c): The equation of the ellipse can be written as

$\frac{(x+1)^2}{1} + \frac{(y-1)^2}{2^2} = 1$ .....(1)

Parametric form of (1) is

$x + 1 = \cos\theta$ and $y - 1 = 2\sin\theta$

$\Rightarrow x = \cos\theta - 1$ and $y = 2\sin\theta + 1$

63. (c): The distance P from the corresponding directrix $= \frac{5}{3} \times 6 = 10$

64. (e) : Length of latus rectum $= 4\sqrt{3}$

$\Rightarrow \quad \frac{2b^2}{a} = 4\sqrt{3}$ ...(1)

and length of transverse axis $= 2\sqrt{3}$

$\Rightarrow \quad 2a = 2\sqrt{3} \Rightarrow a = \sqrt{3}$

Putting the value of a in (1), we get

$\frac{2b^2}{\sqrt{3}} = 4\sqrt{3} \Rightarrow 2b^2 = 12 \Rightarrow b^2 = 6$

Equation of hyperbola is $\frac{x^2}{3} - \frac{y^2}{6} = 1$

65. (d) : Given $e = \frac{5}{4}$

So, focus : $(ae, 0) = \left(\frac{5a}{4}, 0\right)$

$2x + 3y - 6 = 0$ passes through $\left(\frac{5a}{4}, 0\right)$

$\Rightarrow \quad 2\left(\frac{5a}{4}\right) + 3(0) - 6 = 0$

$\Rightarrow \quad \frac{5a}{2}=6 \Rightarrow a=\frac{12}{5}$

Length of transverse axis $= 2a=\frac{24}{5}$

66. (a) : Length of transverse axis of hyperbola $= 2\cos\alpha$

$\Rightarrow \quad 2a = 2\cos\alpha \Rightarrow a=\cos\alpha$ ...(1)

Required equation of ellipse can be written as

$\Rightarrow \quad \frac{x^2}{4^2}+\frac{y^2}{3^2}=1$

Foci of ellipse $= (\pm\sqrt{7},0) =$ foci of hyperbola

$\Rightarrow \quad e\cos\alpha=\pm\sqrt{7} \Rightarrow e=\pm\frac{\sqrt{7}}{\cos\alpha}$ ...(2)

Now, $e=\frac{b^2}{a^2}+1 \Rightarrow \left(\frac{\sqrt{7}}{\cos\alpha}\right)^2=\frac{b^2}{\cos^2\alpha}+1$

$\Rightarrow \quad \frac{7}{\cos^2\alpha}=\frac{b^2}{\cos^2\alpha}+1 \Rightarrow 7=b^2+\cos^2\alpha$

$\Rightarrow b^2=7-\cos^2\alpha$

Required equation of hyperbola is

$$\frac{x^2}{\cos^2\alpha}-\frac{y^2}{7-\cos^2\alpha}=1$$

67. (a) : Here $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}$

$\Rightarrow \quad |\vec{a}|=\sqrt{1^2+2^2+2^2}=3$

Required area $=\frac{1}{2}|\vec{a}\times\vec{b}|=\frac{1}{2}|\vec{a}||\vec{b}|\sin\theta$

$=\frac{1}{2}\times 3\times 5\times \sin\frac{\pi}{6}$

$=\frac{1}{2}\times 3\times 5\times \frac{1}{2}=\frac{15}{4}$

68. (e) : $\cos 60^\circ=\frac{(\vec{a}+\vec{b})\cdot\vec{a}}{|\vec{a}+\vec{b}|\cdot|\vec{a}|}$

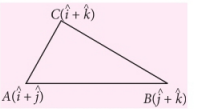
$\Rightarrow \quad \frac{1}{2}=\frac{\vec{a}^2+\vec{a}\cdot\vec{b}}{|\vec{a}+\vec{b}|\cdot|\vec{a}|}=\frac{|\vec{a}|^2}{|\vec{a}+\vec{b}|\cdot|\vec{a}|}$

$\Rightarrow \quad \frac{1}{2}=\frac{|\vec{a}|}{|\vec{a}+\vec{b}|}$, where $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2$

$\Rightarrow \quad \frac{1}{2}=\frac{|\vec{a}|}{\sqrt{|\vec{a}|^2+|\vec{b}|^2}} \Rightarrow \vec{b}=\sqrt{3}|\vec{a}|$

69. (e) : $\overrightarrow{AC}=\hat{k}-\hat{j}$

$\overrightarrow{AB}=\hat{k}-\hat{i}$



$\cos\theta=\frac{1-0-0}{\sqrt{2}\cdot\sqrt{2}}=\frac{1}{2}$

$\Rightarrow \quad \theta=\frac{\pi}{3}$

70. (a) : $|\vec{a}-\vec{b}|^2=|\vec{a}|^2-2\vec{a}\cdot\vec{b}+|\vec{b}|^2$

$\Rightarrow \quad 7=14-2|\vec{b}|^2+|\vec{b}|^2$

$\Rightarrow \quad |\vec{b}|^2=7 \Rightarrow |\vec{b}|=\sqrt{7}$

71. (a) :

$|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2[\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}]$

$\vec{a}\perp(\vec{b}+\vec{c}) \Rightarrow \vec{a}\cdot(\vec{b}+\vec{c})=0$ ...(1)

Similarly, $\vec{b}\cdot(\vec{c}+\vec{a})=0$ ...(2)

and $\vec{c}\cdot(\vec{a}+\vec{b})=0$ ...(3)

Adding (1), (2) and (3), we get

$2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow \quad |\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2$

72. (b) :

$|\vec{u}+\vec{v}+\vec{w}|^2=|\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2+2[\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{u}\cdot\vec{w}]$

$0=9+16+25+2[\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{u}\cdot\vec{w}]$

$\Rightarrow \quad [\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{u}]=-25$

73. (a) : $|\lambda\cdot(3\hat{i}+2\hat{j}-6\hat{k})|=1$

$\Rightarrow \quad |\lambda|\sqrt{9+4+36}=1$

$\Rightarrow \quad |\lambda|\sqrt{49}=1 \Rightarrow |\lambda|=\pm\frac{1}{7}$

74. (b) : Required vector $=3(l\hat{i}+m\hat{j}+n\hat{k})$

where $l^2+m^2+n^2=1$

$\Rightarrow \frac{4}{9}+\frac{a^2}{9}+\frac{4}{9}=1 \Rightarrow \frac{a^2}{9}+\frac{8}{9}=1$

$\Rightarrow \quad a^2=1 \Rightarrow a=1$

Required vector $=3\left(\frac{2}{3}\hat{i}-\frac{1}{3}\hat{j}+\frac{2}{3}\hat{k}\right)$

$=2\hat{i}-\hat{j}+2\hat{k}$

75. (e) : Midpoint of $PQ=\left(\frac{4-1}{2},\frac{5+2}{2},\frac{-10+1}{2}\right)$

$=\left(\frac{3}{2},\frac{7}{2},\frac{-9}{2}\right)$

Position vector $= \left(\frac{3}{2}\hat{i} + \frac{7}{2}\hat{j} - \frac{9}{2}\hat{k}\right)$

Direction ratios of the normal is (5, 3, –11)

Normal vector $\vec{N}$ perpendicular to the plane is $\vec{N} = 5\hat{i} + 3\hat{j} - 11\hat{k}$

Required equation of plane is

$$\left[\vec{r} - \left(\frac{3}{2}\hat{i} + \frac{7}{2}\hat{j} - \frac{9}{2}\hat{k}\right)\right] \cdot 5\hat{i} + 3\hat{j} - 11\hat{k} = 0$$

$$\Rightarrow \quad \vec{r} \cdot (5\hat{i} + 3\hat{j} - 11\hat{k}) - \left(\frac{15}{2} + \frac{21}{2} + \frac{99}{2}\right) = 0$$

$$\Rightarrow \quad \vec{r} \cdot (5\hat{i} + 3\hat{j} - 11\hat{k}) = \frac{135}{2}$$

76. (d) : Equation of first line is

$$\frac{x-1}{1} = \frac{y + \frac{3}{2}}{\left(\frac{3}{2}\right)} = \frac{z+5}{2}$$

Equation of second line is

$$\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$$

$$\cos\theta = \frac{1 \cdot 3 + \left(\frac{3}{2}\right)(-2) + 2 \cdot 0}{\sqrt{(1)^2 + \left(\frac{3}{2}\right)^2 + (2)^2}\sqrt{(3)^2 + (-2)^2 + 0^2}}$$

$\cos\theta = 0 \Rightarrow \theta = 90°$

77. (a) : Let d.r.'s of the line of intersection of the plane be *a, b, c*

$a - 2b - c = 0$ ...(1)

$a + b + 3c = 0$ ...(2)

Solving (1) and (2), we get

$$\frac{a}{-5} = \frac{b}{-4} = \frac{c}{3} \Rightarrow a = -5k, b = -4k, c = 3k$$

$$\Rightarrow \quad \frac{x-2}{-5} = \frac{y-3}{-4} = \frac{z-1}{3}$$

78. (c): Direction ratios of the line are (3, –1, –4) direction cosines are :

$$\left(\frac{3}{\sqrt{3^2+1^2+4^2}}, \frac{-1}{\sqrt{3^2+1^2+4^2}}, \frac{-4}{\sqrt{3^2+1^2+4^2}}\right)$$

$$\left(\frac{3}{\sqrt{26}}, \frac{-1}{\sqrt{26}}, \frac{-4}{\sqrt{26}}\right)$$

79. (c): d.r.'s of the normal is (2, –1, 2)

d.r.'s of *z*-axis is (0, 0, 1)

$$\cos\theta = \left(\frac{2 \cdot 0 + (-1) \cdot 0 + (2)(1)}{\sqrt{2^2+1^2+2^2}\sqrt{0^2+0^2+1}}\right) = \frac{2}{3}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

80. (d) : Required equation of foot of perpendicular drawn from origin to the given plane is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = -\frac{(ax_1 + by_1 + cz_1)}{a^2+b^2+c^2}$$

$$\frac{x-0}{2} = \frac{y-0}{-3} = \frac{z-0}{4} = \frac{29}{29} = 1$$

$\Rightarrow \quad x = 2, y = -3, z = 4$

81. (b) : Distance between the *x*-axis and the point (3, 12, 5) is $\sqrt{12^2 + 5^2} = \sqrt{169} = 13$

82. (d) : $\Sigma x_i - 45 = 9 \Rightarrow \Sigma x_i = 54$

Now, $\Sigma x_i^2 - 10 \times 54 + 25 \times 9 = 45$

$\Rightarrow \quad \Sigma x_i^2 = 360$

$$\sigma = \sqrt{\frac{360}{9} - \left(\frac{54}{9}\right)^2} = 2$$

83. (e) : Total number of outcomes = 36

Sample space of number of outcomes that the sum of the numbers is more than 5 is

(1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4,) (3, 5), (3, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) = 26

$$\text{Required probability} = \frac{26}{36} = \frac{13}{18}$$

84. (d) : $P(A \cup B)' = P(A' \cap B')$

$= P(A')\ P(B') = [\ 1 - P(A)]\ P(B')$

85. (d) : $\bar{X} = \frac{9 + 16 + 23 + 30 + 37 + 44 + 51}{7} = 30$

x_i	$x_i - \bar{X}$	$(x_i - \bar{X})^2$
9	−21	441
16	−14	196
23	−7	49
30	0	0
37	7	49
44	14	196
51	21	441
		$\Sigma(x_i - \bar{X})^2 = 1372$

$\text{Variance} = \frac{1}{n}\Sigma(x_i - \bar{X})^2 = \frac{1}{7} \times 1372 = 196$

$\sigma = \sqrt{\text{Variance}} = 14$

86. (c): $\lim_{x\to 3} \frac{x^5 - 3^5}{x^8 - 3^8} = \lim_{x\to 3} \frac{x^5 - 3^5}{(x-3)} \cdot \left(\frac{x-3}{x^8 - 3^8}\right)$

$= \frac{5 \cdot 3^4}{8 \cdot 3^7} = \frac{5}{8 \cdot 3^3} = \frac{5}{216}$

87. (e) : $h(x) = \lim_{x\to 1} \frac{(x^5 - 1)(x^3 + 1)}{(x^2 - 1)(x^2 - x + 1)}$

$= \lim_{x\to 1} \frac{(x^5 - 1)(x + 1)(x^2 - x + 1)}{(x + 1)(x - 1)(x^2 - x + 1)}$

$= \lim_{x\to 1} \frac{x^5 - 1}{x - 1} = 5 \cdot (1)^4 = 5$

88. (a) : $\lim_{x\to 0} \frac{\log(1 + 3x^2)}{x(e^{5x} - 1)}$

$= \lim_{x\to 0} \frac{\frac{\log(1 + 3x^2)}{3x^2} \cdot 3x}{\left(\frac{e^{5x} - 1}{5x}\right)5x} = \frac{3}{5}$

89. (a) : We have $f(x) = \frac{x+2}{3x-1}$

$f(f(x)) = f\left(\frac{x+2}{3x-1}\right) = \frac{\left(\frac{x+2}{3x-1}\right) + 2}{3\left(\frac{x+2}{3x-1}\right) - 1}$

$= \frac{x + 2 + 2(3x - 1)}{3(x + 2) - (3x - 1)}$

$= \frac{x + 2 + 6x - 2}{3x + 6 - 3x + 1} = \frac{7x}{7} = x$

90. (c): $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$\lim_{x\to 2^-} ax + 3 = \lim_{x\to 2^+} a^2 x - 1$

$\Rightarrow \quad 2a + 3 = 2a^2 - 1 \quad \Rightarrow \quad 2a^2 - 2a - 4 = 0$

$\Rightarrow \quad a^2 - a - 2 = 0 \quad \Rightarrow \quad (a + 1)(a - 2) = 0$

$\Rightarrow \quad a = -1$ and 2

91. (c): We have $|f(x) - f(y)|^2 \le |x - y|^3$ ...(1)

$|f(y) - f(x)|^2 \le |y - x|^3$ ...(2)

If $x > y$, (2) is wrong

If $x < y$, (1) is wrong

So, $f(x) = c$, a constant

$\Rightarrow \quad f'(x) = 0$

92. (d) : $f(x) = \frac{1}{2}\int_1^x (1 - \cos t)\, dt$

$= \frac{1}{2}[t - \sin t]_1^x = \frac{1}{2}[x - \sin x - 1 + \sin 1]$

$f'(x) = \frac{1}{2}[1 - \cos x] \quad \Rightarrow \quad f'(\pi) = \frac{1}{2}[1 + 1] = 1$

93. (e) : We have $y = f(x^2 + 2)$ ...(1)

Differentiating (1) w.r.t. x, we get

$\frac{dy}{dx} = f'(x^2 + 2)\,(2x)$

$\left.\frac{dy}{dx}\right|_{\text{at } x = 1} = 2f'(3) = 2 \times 5 = 10$

94. (c): We have $f(x) = x^2 + bx + 7$ ...(1)

Differentiating (1) w.r.t. x, we get

$f'(x) = 2x + b$ ....(2)

Now, $\quad f'(5) = 2f'\left(\frac{7}{2}\right)$

$10 + b \Rightarrow = 2(7 + b) \quad (\because$ using (2))

$\Rightarrow \quad 10 + b = 14 + 2b \Rightarrow \quad b = -4$

95. (b) : We have $x = \sin t$ ...(1)

and $y = \tan t$ ...(2)

Differentiating (1) and (2) w.r.t. t, we get

$\frac{dx}{dt} = \cos t$ and $\frac{dy}{dt} = \sec^2 t$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\sec^2 t}{\cos t}=\frac{1}{\cos^3 t}$$

96. (d) : We have $x = a\cos^3\theta$ ...(1)

and $y = a\sin^3\theta$ ...(2)

Differentiating (1) and (2) w.r.t. θ, we get

$$\frac{dx}{d\theta}=-3a\cos^2\theta\sin\theta \text{ and } \frac{dy}{d\theta}=3a\sin^2\theta\cos\theta$$

$$\frac{dy}{dx}=\frac{dy}{d\theta}\Big/\frac{dx}{d\theta}=\frac{-3a\sin^2\theta\cos\theta}{3a\cos^2\theta\sin\theta}$$

$$=-\frac{\sin\theta}{\cos\theta}=-\tan\theta$$

$$\text{Now, } 1+\left(\frac{dy}{dx}\right)^2=1+\tan^2\theta=\sec^2\theta$$

97. (c): We have $y=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ ...(1)

Putting $x = \sin\theta$ in (1), we get

$y = \sin^{-1}(2\sin\theta\cos\theta)$

$\Rightarrow\ y = \sin^{-1}(\sin 2\theta) = 2\theta\ \Rightarrow\ y = 2\sin^{-1}x$ ...(2)

Differentiating (2) w.r.t. x, we get

$$\frac{dy}{dx}=\frac{2}{\sqrt{1-x^2}}$$

98. (b) : We have $y^2 = 4x + 5$ ...(1)

Differentiating (1) w.r.t. x, we get

$$2y\frac{dy}{dx}=4\ \Rightarrow\ \frac{dy}{dx}=\frac{2}{y} \quad ...(2)$$

Also, $2x - y + 5 = 0$ ...(3)

Differentiating (3) w.r.t. x, we get

$$2-\frac{dy}{dx}=0\ \Rightarrow\ \frac{dy}{dx}=2 \quad ...(4)$$

From (2) and (4), $y = 1$

Putting the value of y in (1), we get

$1 = 4x + 5\ \Rightarrow\ x = -1$

Point of contact is $(-1, 1)$

99. (a) : We have, $f(x) = 2x^3 - 15x^2 + 36x + 6$ ...(1)

Differentiating (1) w.r.t. x, we get

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6) = 6(x - 2)(x - 3) < 0$

$\Rightarrow\ x \in (2, 3)$

100.(c): The given curve is $y^2e^{xy} = 9e^{-3}x^2$

Taking log both sides, we get

$2\log y + xy = \log 9 - 3 + 2\log x$ .....(1)

Differentiating (1) w.r.t x, we get

$$\frac{2}{y}\frac{dy}{dx}+x\frac{dy}{dx}+y=\frac{2}{x}$$

$$\frac{dy}{dx}\left(\frac{2}{y}+x\right)+y=\frac{2}{x}$$

$$\text{Now } \left.\frac{dy}{dx}\right|_{(-1,3)}\left(\frac{2}{3}+(-1)\right)+3=\frac{2}{-1}$$

$$\Rightarrow\ \left.\frac{dy}{dx}\right|_{(-1,3)}\left(\frac{-1}{3}\right)=-5\ \Rightarrow\ \left.\frac{dy}{dx}\right|_{(-1,3)}=15$$

101.(a) : Let $V = \pi r^2 h$ .....(1)

Differentiating (1) w.r.t. t, we get

$$\frac{dV}{dt}=\pi\left[r^2\frac{dh}{dt}+h\cdot 2r\frac{dr}{dt}\right]=0 \quad ..... (2)$$

We have $\frac{dr}{dt} = 5$ cm/min, $r = 5$ cm, $h = 3$ cm

Substituting the values in (2), we get

$$25\cdot\frac{dh}{dt}+30(5)=0\ \Rightarrow\ \frac{dh}{dt}=\frac{-150}{25}$$

$$\Rightarrow\ \frac{dh}{dt}=-6$$

102. (e) :

103. (a): We have $y=x^2-\frac{1}{x^2}$ ...(1)

Differentiating (1) w.r.t. x, we get

$$\frac{dy}{dx}=2x+\frac{2}{x^3}=-2+(-2)=-4$$

$$\text{Slope of normal}=-\left(\frac{1}{\frac{dy}{dx}}\right)=\frac{1}{4}$$

104.(b) : Minimum value of $\sin x + \cos x$ is

$$-\sqrt{1^2+1^2}=-\sqrt{2}$$

105.(c): $I=\int\frac{dx}{x^2(x^4+1)^{3/4}}=\int\frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

Put $1+\frac{1}{x^4}=t \Rightarrow \frac{-4}{x^5}dx=dt$

$I=-\frac{1}{4}\int\frac{dt}{t^{3/4}}=-\frac{1}{4}\frac{t^{1/4}}{1/4}+C=-\left(1+\frac{1}{x^4}\right)^{1/4}+C$

106.(e) : We have $I=\int\frac{(1+x)e^x}{\sin^2(xe^x)}dx$

Let $xe^x=t \Rightarrow (xe^x+e^x)dx=dt$

So, $I=\int\frac{dt}{\sin^2 t}=\int \text{cosec}^2 t\,dt$

$=-\cot t+C=-\cot(xe^x)+C$

107.(b) : The given integral can be written as

$\int e^x\left(\frac{1}{(1+x)}-\frac{1}{(1+x)^2}\right)dx=\frac{e^x}{(x+1)}+c$

108.(c): $e^x(\sin^2 x+2\sin x\cos x)\,dx$

$=e^x\sin^2 x+c$

109.(e) : $I=\int\sqrt{1+\cos x}\,dx$

$I=\int\sqrt{2\cos^2(x/2)}\,dx=\sqrt{2}\int\cos\frac{x}{2}\,dx$

$=\frac{\sqrt{2}\sin\left(\frac{x}{2}\right)}{1/2}+C=2\sqrt{2}\sin\frac{x}{2}+C$

110.(a) : We have $I=\int\frac{\sqrt{x^2-1}}{x}\,dx$

Putting $x=\sec\theta, \Rightarrow dx=\sec\theta\tan\theta d\theta$

$I=\int\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}\sec\theta\tan\theta\,d\theta$

$I=\int\frac{\tan\theta\sec\theta\tan\theta}{\sec\theta}d\theta$

$=\int\tan^2\theta\,d\theta=\int(\sec^2\theta-1)\,d\theta$

$=\tan\theta-\theta+C=\sqrt{x^2-1}-\sec^{-1}x+C$

111.(c): Let $I=\frac{\sqrt{5+x^2}}{x^4}dx$

Put $x=\sqrt{5}\tan\theta \Rightarrow dx=\sqrt{5}\sec^2\theta\,d\theta$

$I=\int\frac{\sqrt{5+5\tan^2\theta}}{25\tan^4\theta}\cdot\sqrt{5}\sec^2\theta\,d\theta$

$=\int\sqrt{5}\frac{\sqrt{5}\sec\theta}{25\tan^4\theta}\cdot\sec^2\theta\,d\theta=\frac{1}{5}\int\frac{\cos\theta}{\sin^4\theta}d\theta$

Put $\sin\theta=t \Rightarrow \cos\theta d\theta=dt$

$\frac{1}{5}\int\frac{dt}{t^4}=\frac{1}{5}\left(\frac{t^{-3}}{-3}\right)=-\frac{1}{15}\frac{1}{t^3}+C$

$=-\frac{1}{15}\frac{1}{\sin^3\theta}+C=-\frac{1}{15}\cdot\frac{1}{\left(\sqrt{\frac{x^2}{x^2+5}}\right)^3}+C$

$=-\frac{1}{15}\left(\sqrt{\frac{x^2+5}{x^2}}\right)^3+C=-\frac{1}{15}\left(1+\frac{5}{x^2}\right)^{3/2}+C$

112.(a) : Let $I=\int_0^1\frac{dx}{e^x+e} \Rightarrow I=\int_0^1\frac{e^{-x}dx}{1+ee^{-x}}$

Let $1+ee^{-x}=t \Rightarrow -ee^{-x}dx=dt$

$\Rightarrow e^{-x}dx=-\frac{1}{e}dt$

Now, for $x=0$, $t=1+e$ and $x=1$, $t=2$

$I=\frac{-1}{e}\int_{1+e}^{2}\frac{dt}{t}$

$=\frac{-1}{e}[\log t]_{1+e}^{2}=\frac{-1}{e}[\log 2-\log(1+e)]$

$=\frac{1}{e}[\log(1+e)-\log 2]=\frac{1}{e}\log\left(\frac{1+e}{2}\right)$

113.(c): Required area $=\int_0^1(e^x-e^{-x})\,dx$

$=(e^x+e^{-x})]_0^1=(e^1+e^{-1})-2=e+\frac{1}{e}-2$

114.(b) : Let $1+\log x=t \Rightarrow \frac{1}{x}dx=dt$

The given integral reduces to

$\frac{1}{3}\int_1^2 t\,dt=\frac{1}{3}\left(\frac{t^2}{2}\right)_1^2=\frac{1}{3}\left(\frac{4}{2}-\frac{1}{2}\right)=\frac{1}{3}\left(2-\frac{1}{2}\right)=\frac{1}{2}$

115.(c): Put $x^4 = t \Rightarrow 4x^3\,dx = dt$

The given integral reduces to

$$\frac{1}{4}\int_0^1 \frac{dt}{1+t^2} = \frac{1}{4}(\tan^{-1} t)_0^1 = \frac{1}{4}\left(\frac{\pi}{4}\right) = \frac{\pi}{16}$$

116.(c): Let $\log t = x \Rightarrow \frac{1}{t}dt = dx$

$$\int_{\log 2}^{\log 4} x\,dx = \left[\frac{x^2}{2}\right]_{\log 2}^{\log 4}$$

$$= \frac{1}{2}\left[(\log 2^2)^2 - (\log 2)^2\right]$$

$$= \frac{1}{2}\left[(\log 2^2 - \log 2)(\log 2^2 + \log 2)\right]$$

$$= \frac{1}{2}\left[(\log 2)(3\log 2)\right] = \frac{3}{2}(\log 2)^2$$

117.(b) : $kx\,dy - y^2dy = x^2dx - ky\,dx$

$k[x\,dy + y\,dx] = x^2dx + y^2dy$

$$\Rightarrow \quad k\,d(xy) = \frac{1}{3}d(x^3 + y^3)$$

$$\Rightarrow \quad x^3 + y^3 = 3kxy + c$$

118.(b) : $\frac{dy}{dx} = e^x + 1 \Rightarrow dy = (e^x + 1)\,dx$

$$\Rightarrow \quad y = e^x + x + C$$

119.(e): The given differential equation becomes

$$\left(y - \frac{d^2y}{dx^2}\right) = \left(\frac{dy}{dx}\right)^{3/2} \Rightarrow \left(y - \frac{d^2y}{dx^2}\right)^2 = \left(\frac{dy}{dx}\right)^3$$

Order and degree of the differential equation is 2, 2.

120.(a) : The given differential equation can be written as $\frac{dy}{dx} + 2y\cot x = \text{cosec}\,x$

$$\text{I.F.} = e^{\int 2\cot x\,dx} = e^{2\log\sin x} = \sin^2 x$$

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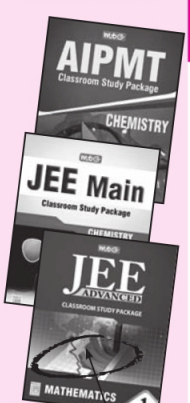

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SOLVED PAPER 2014

WB-JEE

Q.1 to Q.60 carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of 1/3 marks.

1. Let the equation of an ellipse be $\frac{x^2}{144}+\frac{y^2}{25}=1$.

Then the radius of the circle with centre $(0,\sqrt{2})$ and passing through the foci of the ellipse is

(a) 9 (b) 7 (c) 11 (d) 5

2. If $y = 4x + 3$ is parallel to a tangent to the parabola $y^2 = 12x$, then its distance from the normal parallel to the given line is

(a) $\frac{213}{\sqrt{17}}$ (b) $\frac{219}{\sqrt{17}}$ (c) $\frac{211}{\sqrt{17}}$ (d) $\frac{210}{\sqrt{17}}$

3. In a ΔABC, $\tan A$ and $\tan B$ are roots of $pq(x^2 + 1) = r^2x$. Then ΔABC is

(a) a right angled triangle
(b) an acute angled triangle
(c) an obtuse angled triangle
(d) an equilateral triangle

4. Let the number of elements of the sets A and B be p and q respectively. Then the number of the relations from the set A to the set B is

(a) 2^{p+q} (b) 2^{pq} (c) $p + q$ (d) pq

5. The function $f(x)=\frac{\tan\left\{\pi\left[x-\frac{\pi}{2}\right]\right\}}{2+[x]^2}$, where $[x]$ denotes the greatest integer $\le x$, is

(a) continuous for all values of x
(b) discontinuous at $x = \pi/2$
(c) not differentiable for some values of x
(d) discontinuous at $x = -2$

6. Let z_1, z_2 be two fixed complex numbers in the Argand plane and z be an arbitrary point satisfying $|z - z_1| + |z - z_2| = 2|z_1 - z_2|$. Then the locus of z will be

(a) an ellipse
(b) a straight line joining z_1 and z_2
(c) a parabola
(d) a bisector of the line segment joining z_1 and z_2

7. Let $S=\frac{2}{1}{}^nC_0+\frac{2^2}{2}{}^nC_1+\frac{2^3}{3}{}^nC_2+\ldots+\frac{2^{n+1}}{n+1}{}^nC_n$. Then S equals

(a) $\frac{2^{n+1}-1}{n+1}$ (b) $\frac{3^{n+1}-1}{n+1}$

(c) $\frac{3^n-1}{n}$ (d) $\frac{2^n-1}{n}$

8. Out of 7 consonants and 4 vowels, the number of words (not necessarily meaningful) that can be made, each consisting of 3 consonants and 2 vowels, is

(a) 24800 (b) 25100
(c) 25200 (d) 25400

9. The remainder obtained when $1! + 2! + 3! + \ldots + 11!$ is divided by 12 is

(a) 9 (b) 8
(c) 7 (d) 6

10. Let S denote the sum of the infinite series $1+\frac{8}{2!}+\frac{21}{3!}+\frac{40}{4!}+\frac{65}{5!}+\ldots$. Then

(a) $S < 8$ (b) $S > 12$
(c) $8 < S < 12$ (d) $S = 8$

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11. For every real number x, let

$$f(x)=\frac{x}{1!}+\frac{3}{2!}x^2+\frac{7}{3!}x^3+\frac{15}{4!}x^4+\ldots$$

Then the equation $f(x)=0$ has
(a) no real solution
(b) exactly one real solution
(c) exactly two real solutions
(d) infinite number of real solutions

12. The coefficient of x^3 in the infinite series expansion of $\frac{2}{(1-x)(2-x)}$, for $|x|<1$, is
(a) –1/16 (b) 15/8 (c) –1/8 (d) 15/16

13. If α, β are the roots of the quadratic equation $x^2+px+q=0$, then the values of $\alpha^3+\beta^3$ and $\alpha^4+\alpha^2\beta^2+\beta^4$ are respectively
(a) $3pq-p^3$ and $p^4-3p^2q+3q^2$
(b) $-p(3q-p^2)$ and $(p^2-q)(p^2+3q)$
(c) $pq-4$ and p^4-q^4
(d) $3pq-p^3$ and $(p^2-q)(p^2-3q)$

14. A fair six-faced die is rolled 12 times. The probability that each face turns up twice is equal to

(a) $\frac{12!}{6!6!6^{12}}$ (b) $\frac{2^{12}}{2^6 6^{12}}$

(c) $\frac{12!}{2^6 6^{12}}$ (d) $\frac{12!}{6^2 6^{12}}$

15. Let $f(x)$ be a differentiable function in $[2, 7]$. If $f(2)=3$ and $f'(x)\le 5$ for all x in $(2, 7)$, then the maximum possible value of $f(x)$ at $x=7$ is
(a) 7 (b) 15 (c) 28 (d) 14

16. The value of $\tan\frac{\pi}{5}+2\tan\frac{2\pi}{5}+4\cot\frac{4\pi}{5}$ is

(a) $\cot\frac{\pi}{5}$ (b) $\cot\frac{2\pi}{5}$

(c) $\cot\frac{4\pi}{5}$ (d) $\cot\frac{3\pi}{5}$

17. Let R be the set of all real numbers and $f: R\to R$ be given by $f(x)=3x^2+1$. Then the set $f^{-1}([1, 6])$ is

(a) $\left\{-\sqrt{\frac{5}{3}}, 0, \sqrt{\frac{5}{3}}\right\}$ (b) $\left\{-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right\}$

(c) $\left\{-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right\}$ (d) $\left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

18. The area of the region bounded by the curves $y=x^2$ and $x=y^2$ is
(a) 1/3 (b) 1/2 (c) 1/4 (d) 3

19. The point on the parabola $y^2=64x$ which is nearest to the line $4x+3y+35=0$ has coordinates
(a) (9, –24) (b) (1, 81)
(c) 4, –16) (d) (–9, –24)

20. The equation of the common tangent with positive slope to the parabola $y^2=8\sqrt{3}x$ and hyperbola $4x^2-y^2=4$ is
(a) $y=\sqrt{6}x+\sqrt{2}$ (b) $y=\sqrt{6}x-\sqrt{2}$
(c) $y=\sqrt{3}x+\sqrt{2}$ (d) $y=\sqrt{3}x-\sqrt{2}$

21. Let p, q be real numbers. If α is the root of $x^2+3p^2x+5q^2=0$, β is a root of $x^2+9p^2x+15q^2=0$ and $0<\alpha<\beta$, then the equation $x^2+6p^2x+10q^2=0$ has a root γ that always satisfies
(a) $\gamma=\alpha/4+\beta$ (b) $\beta<\gamma$
(c) $\gamma=\alpha/2+\beta$ (d) $\alpha<\gamma<\beta$

22. The value of the sum $({}^nC_1)^2+({}^nC_2)^2+({}^nC_3)^2+\ldots+({}^nC_n)^2$ is
(a) $({}^{2n}C_n)^2$ (b) ${}^{2n}C_n$
(c) ${}^{2n}C_n+1$ (d) ${}^{2n}C_n-1$

23. Ram is visiting a friend. Ram knows that his friend has 2 children and 1 of them is a boy. Assuming that a child is equally likely to be a boy or a girl, then the probability that the other child is a girl, is
(a) 1/2 (b) 1/3 (c) 2/3 (d) 7/10

24. Let $n\ge 2$ be an integer,

$$A=\begin{pmatrix}\cos(2\pi/n) & \sin(2\pi/n) & 0\\ -\sin(2\pi/n) & \cos(2\pi/n) & 0\\ 0 & 0 & 1\end{pmatrix}$$

and I is the identity martix of order 3. Then
(a) $A^n=I$ and $A^{n-1}\ne I$
(b) $A^m\ne I$ for any positive integer m
(c) A is not invertible
(d) $A^m=0$ for a positive integer m

25. Let I denote the 3×3 identity matrix and P be a matrix obtained by rearranging the columns of I. Then
(a) there are six distinct choices for P and $\det(P) = 1$
(b) there are six distinct choices for P and $\det(P) = \pm 1$
(c) there are more than one choices for P and some of them are not invertible
(d) there are more than one choices for P and $P^{-1} = I$ in each choice

26. The sum of the series $\sum_{n=1}^{\infty} \sin\left(\frac{n!\pi}{720}\right)$ is
(a) $\sin\left(\frac{\pi}{180}\right) + \sin\left(\frac{\pi}{360}\right) + \sin\left(\frac{\pi}{540}\right)$
(b) $\sin\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{30}\right) + \sin\left(\frac{\pi}{120}\right) + \sin\left(\frac{\pi}{360}\right)$
(c) $\sin\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{30}\right) + \sin\left(\frac{\pi}{120}\right) + \sin\left(\frac{\pi}{360}\right) + \left(\frac{\pi}{720}\right)$
(d) $\sin\left(\frac{\pi}{180}\right) + \sin\left(\frac{\pi}{360}\right)$

27. Let α, β be the roots of $x^2 - x - 1 = 0$ and $S_n = \alpha^n + \beta^n$, for all integers $n \geq 1$. Then for every integer $n \geq 2$,
(a) $S_n + S_{n-1} = S_{n+1}$
(b) $S_n - S_{n-1} = S_{n+1}$
(c) $S_{n-1} = S_{n+1}$
(d) $S_n + S_{n-1} = 2S_{n+1}$

28. In a ΔABC, a, b, c are the sides of the triangle opposite to the angles A, B, C respectively. Then the value of
$a^3 \sin(B - C) + b^3 \sin(C - A) + c^3 \sin(A - B)$
is equal to
(a) 0 (b) 1 (c) 3 (d) 2

29. In the Argand plane, the distinct roots of $1 + z + z^3 + z^4 = 0$ (z is a complex number) represent vertices of
(a) a square
(b) an equilateral triangle
(c) a rhombus
(d) a rectangle

30. The number of digits in 20^{301} (given $\log_{10} 2 = 0.3010$) is
(a) 602 (b) 301 (c) 392 (d) 391

31. If $\sqrt{y} = \cos^{-1} x$, then it satisfies the differential equation $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = c$, where c is equal to
(a) 0 (b) 3 (c) 1 (d) 2

32. The integrating factor of the differential equation $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$ is
(a) $\tan^{-1} x$ (b) $1 + x^2$
(c) $e^{\tan^{-1} x}$ (d) $\log_e(1 + x^2)$

33. The solution of the equation $\log_{101} \log_7(\sqrt{x+7} + \sqrt{x}) = 0$ is
(a) 3 (b) 7 (c) 9 (d) 49

34. If α, β are the roots of $ax^2 + bx + c = 0$ ($a \neq 0$) and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0$ ($p \neq 0$) then the ratio of the squares of their discriminants is
(a) $a^2 : p^2$ (b) $a : p^2$
(c) $a^2 : p$ (d) $a : 2p$

35. Let $f(x) = 2x^2 + 5x + 1$. If we write $f(x)$ as $f(x) = a(x+1)(x-2) + b(x-2)(x-1) + c(x-1)(x+1)$ for real numbers a, b, c, then
(a) there are infinite number of choices for a, b, c
(b) only one choice for a but infinite number of choices for b and c
(c) exactly one choice for each of a, b, c
(d) more than one but finite number of choices for a, b, c

36. Let $f(x) = x + 1/2$. Then the number of real values of x for which the three unequal terms $f(x), f(2x), f(4x)$ are in H.P. is
(a) 1 (b) 0 (c) 3 (d) 2

37. The function $f(x) = x^2 + bx + c$, where b and c real constants, describes
(a) one-to-one mapping
(b) onto mapping
(c) not one-to-one but onto mapping
(d) neither one-to-one nor onto mapping

38. Suppose that the equation $f(x) = x^2 + bx + c = 0$ has two distinct real roots α and β. The angle between the tangent to the curve $y = f(x)$ at the point $\left(\frac{\alpha+\beta}{2}, f\left(\frac{\alpha+\beta}{2}\right)\right)$ and the positive direction of the x-axis is

(a) 0° (b) 30° (c) 60° (d) 90°

39. The solution of the differential equation $y\frac{dy}{dx} = x\left[\frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)} + \frac{y^2}{x^2}\right]$ is (where c is a constant)

(a) $\phi\left(\frac{y^2}{x^2}\right) = cx$ (b) $x\phi\left(\frac{y^2}{x^2}\right) = c$

(c) $\phi\left(\frac{y^2}{x^2}\right) = cx^2$ (d) $x^2\phi\left(\frac{y^2}{x^2}\right) = c$

40. Let $f(x)$ be a differentiable function and $f'(4) = 5$. Then $\lim_{x\to 2}\frac{f(4) - f(x^2)}{x-2}$ equals

(a) 0 (b) 5 (c) 20 (d) −20

41. The value of $\lim_{x\to 0}\frac{\int_0^{x^2}\cos(t^2)dt}{x\sin x}$ is

(a) 1 (b) −1 (c) 2 (d) $\log_e 2$

42. The range of the function $y = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ is

(a) $[0, \sqrt{3/2}]$ (b) $[0, 1]$

(c) $[0, 3/\sqrt{2}]$ (d) $[0, \infty)$

43. There is a group of 265 persons who like either singing or dancing or painting. In this group 200 like singing, 110 like dancing and 55 like painting. If 60 persons like both singing and dancing, 30 like both singing and painting and 10 like all three activities, then the number of persons who like only dancing and painting is

(a) 10 (b) 20 (c) 30 (d) 40

44. The curve $y = (\cos x + y)^{1/2}$ satisfies the differential equation

(a) $(2y-1)\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + \cos x = 0$

(b) $\frac{d^2y}{dx^2} - 2y\left(\frac{dy}{dx}\right)^2 + \cos x = 0$

(c) $(2y-1)\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2 + \cos x = 0$

(d) $(2y-1)\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + \cos x = 0$

45. Suppose that z_1, z_2, z_3 are three vertices of an equilateral triangle in the Argand plane. Let $\alpha = \frac{1}{2}(\sqrt{3} + i)$ and β be a non-zero complex number. The points $\alpha z_1 + \beta$, $\alpha z_2 + \beta$, $\alpha z_3 + \beta$ will be

(a) the vertices of an equilateral triangle
(b) the vertices of an isosceles triangle
(c) collinear
(d) the vertices of a scalene triangle

46. If $\lim_{x\to 0}\frac{2a\sin x - \sin 2x}{\tan^3 x}$ exists and is equal to 1, then the value of a is

(a) 2 (b) 1 (c) 0 (d) −1

47. If $f(x) = \begin{cases} 2x^2 + 1, & x \le 1 \\ 4x^3 - 1, & x > 1 \end{cases}$, then $\int_0^2 f(x)dx$ is

(a) 47/3 (b) 50/3 (c) 1/3 (d) 47/2

48. The value of $|z|^2 + |z-3|^2 + |z-i|^2$ is minimum when z equals

(a) $2 - \frac{2}{3}i$ (b) $45 + 3i$

(c) $1 + \frac{i}{3}$ (d) $1 - \frac{i}{3}$

49. The number of solution(s) of the equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ is/are

(a) 2 (b) 0 (c) 3 (d) 1

50. The values of λ for which the curve $(7x + 5)^2 + (7y + 3)^2 = \lambda^2 (4x + 3y - 24)^2$ represents a parabola is

(a) $\pm\frac{6}{5}$ (b) $\pm\frac{7}{5}$

(c) $\pm\frac{1}{5}$ (d) $\pm\frac{2}{5}$

51. If $\sin^{-1}\left(\frac{x}{13}\right)+\text{cosec}^{-1}\left(\frac{13}{12}\right)=\frac{\pi}{2}$, then the value of x is

(a) 5 (b) 4 (c) 12 (d) 11

52. The straight lines $x + y = 0$, $5x + y = 4$ and $x + 5y = 4$ form

(a) an isosceles triangle
(b) an equilateral triangle
(c) a scalene triangle
(d) a right angled triangle

53. If $I=\int_0^2 e^{x^4}(x-\alpha)dx=0$, then α lies in the interval

(a) (0, 2) (b) (−1, 0) (c) (2, 3) (d) (−2, −1)

54. If the coefficient of x^8 in $\left(ax^2+\frac{1}{bx}\right)^{13}$ is equal to the coefficient of x^{-8} in $\left(ax-\frac{1}{bx^2}\right)^{13}$, then a and b will satisfy the relation

(a) $ab + 1 = 0$ (b) $ab = 1$
(c) $a = 1 - b$ (d) $a + b = -1$

55. The function $f(x) = a \sin |x| + be^{|x|}$ is differentiable at $x = 0$ when

(a) $3a + b = 0$ (b) $3a - b = 0$
(c) $a + b = 0$ (d) $a - b = 0$

56. If a, b and c are positive numbers in a G.P., then the roots of the quadratic equation $(\log_e a)x^2 - (2\log_e b)x + (\log_e c) = 0$ are

(a) -1 and $\frac{\log_e c}{\log_e a}$ (b) 1 and $-\frac{\log_e c}{\log_e a}$

(c) 1 and $\log_a c$ (d) -1 and $\log_e a$

57. Let R be the set of all real numbers and $f: [-1, 1] \to R$ be defined by $f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$. Then

(a) f satisfies the conditions of Rolle's theorem on [−1, 1]
(b) f satisfies the conditions of Lagrange's Mean Value Theorem on [−1, 1]
(c) f satisfies the conditions of Rolle's theorem on [0, 1]
(d) f satisfies the conditions of Lagrange's Mean Value Theorem on [0, 1]

58. Let z_1 be a fixed point on the circle of radius 1 centered at the origin in the Argand plane and $z_1 \neq \pm 1$. Consider an equilateral triangle inscribed in the circle with z_1, z_2, z_3 as the vertices taken in the counter clockwise direction. Then $z_1z_2z_3$ is equal to

(a) z_1^2 (b) z_1^3 (c) z_1^4 (d) z_1

59. Suppose that $f(x)$ is a differentiable function such that $f'(x)$ is continuous, $f'(0) = 1$ and $f''(0)$ does not exist. Let $g(x) = x f'(x)$. Then

(a) $g'(0)$ does not exist
(b) $g'(0) = 0$
(c) $g'(0) = 1$
(d) $g'(0) = 2$

60. Let $[x]$ denote the greatest integer less than or equal to x for any real number x. Then $\lim_{n\to\infty}\frac{[n\sqrt{2}]}{n}$ is equal to

(a) 0 (b) 2 (c) $\sqrt{2}$ (d) 1

CATEGORY-II

Q.61 to Q.75 carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of 2/3 marks.

61. We define a binary relation ~ on the set of all 3×3 real matrices as $A \sim B$ if and only if there exist invertible matrices P and Q such that $B = PAQ^{-1}$. The binary relation ~ is

(a) neither reflexive nor symmetric
(b) reflexive and symmetric but not transitive
(c) symmetric and transitive but not reflexive
(d) an equivalence relation

62. The minimum value of $2^{\sin x} + 2^{\cos x}$ is

(a) $2^{1-1/\sqrt{2}}$ (b) $2^{1+1/\sqrt{2}}$
(c) $2^{\sqrt{2}}$ (d) 2

63. For any real numbers θ and ϕ, we define $\theta R \phi$ if and only if $\sec^2\theta - \tan^2\phi = 1$. The relation R is
(a) reflexive but not transitive
(b) symmetric but not reflexive
(c) both reflexive and symmetric but not transitive
(d) an equivalence relation

64. A particle starting from a point A and moving with a positive constant acceleration along a straight line reaches another point B in time T. Suppose that the initial velocity of the particle is $u > 0$ and P is the midpoint of the line AB. If the velocity of the particle at point P is v_1 and if the velocity at time $T/2$ is v_2, then
(a) $v_1 = v_2$ (b) $v_1 > v_2$
(c) $v_1 < v_2$ (d) $v_1 = \frac{1}{2}v_2$

65. Let t_n denote the nth term of the infinite series $\frac{1}{1!}+\frac{10}{2!}+\frac{21}{3!}+\frac{34}{4!}+\frac{49}{5!}+\ldots$. Then $\lim_{n\to\infty} t_n$ is
(a) e (b) 0 (c) e^2 (d) 1

66. Let α, β denote the cube roots of unity other than 1 and $\alpha \neq \beta$. Let $s = \sum_{n=0}^{302} (-1)^n \left(\frac{\alpha}{\beta}\right)^n$. Then the value of s is
(a) either -2ω or $-2\omega^2$
(b) either -2ω or $2\omega^2$
(c) either 2ω or $-2\omega^2$
(d) either 2ω or $2\omega^2$

67. The equation of hyperbola whose coordinates of the foci are $(\pm 8, 0)$ and the length of the latus rectum is 24 units, is
(a) $3x^2 - y^2 = 48$ (b) $4x^2 - y^2 = 48$
(c) $x^2 - 3y^2 = 48$ (d) $x^2 - 4y^2 = 48$

68. Applying Lagrange's mean value theorem for a suitable function $f(x)$ in $[0, h]$, we have $f(h) = f(0) + hf'(\theta h)$, $0 < \theta < 1$. Then for $f(x) = \cos x$, the value of $\lim_{h\to 0^+} \theta$ is
(a) 1 (b) 0 (c) 1/2 (d) 1/3

69. Let $X_n = \left\{ z = x + iy : |z^2| \leq \frac{1}{n} \right\}$ for all integers $n \geq 1$. Then $\bigcap_{n=1}^{\infty} X_n$ is
(a) a singleton set (b) not a finite set
(c) an empty set
(d) a finite set with more than one elements

70. Suppose $M = \int_0^{\pi/2} \frac{\cos x}{x+2}\,dx$, $N = \int_0^{\pi/4} \frac{\sin x \cos x}{(x+1)^2}\,dx$.
Then the value of $(M - N)$ equals
(a) $\frac{3}{\pi+2}$ (b) $\frac{2}{\pi-4}$
(c) $\frac{4}{\pi-2}$ (d) $\frac{2}{\pi+4}$

71. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$
(a) is equal to zero
(b) lies between 0 and 3
(c) is a negative number
(d) lies between 3 and 6

72. A student answers a multiple choice question with 5 alternatives of which exactly one is correct. The probability that he knows the correct answer is p, $0 < p < 1$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is
(a) $\frac{3p}{4p+3}$ (b) $\frac{5p}{3p+2}$
(c) $\frac{5p}{4p+1}$ (d) $\frac{4p}{3p+1}$

73. A poker hand consists of 5 cards drawn at random from a well-shuffled pack of 52 cards. Then the probability that the poker hand consists of a pair and a triple of equal face values (for example, 2 sevens and 3 kings or 2 aces and 3 queens, etc.) is
(a) $\frac{6}{4165}$ (b) $\frac{23}{4165}$ (c) $\frac{1797}{4165}$ (d) $\frac{1}{4165}$

74. Let $f(x) = \max\{x + |x|, x - [x]\}$, where $[x]$ denotes the greatest integer $\leq x$. Then the value of $\int_{-3}^{3} f(x)\,dx$ is
(a) 0 (b) 51/2 (c) 21/2 (d) 1

75. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x\log_e x}=\frac{1}{x}$ under the condition $y = 1$ when $x = e$ is

(a) $2y=\log_e x+\frac{1}{\log_e x}$

(b) $y=\log_e x+\frac{2}{\log_e x}$

(c) $y\log_e x=\log_e x+1$

(d) $y=\log_e x+e$

CATEGORY-III

Q.76 to Q.80 carry two marks each, for which one or more than one options may be correct. Making of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question - irrespective of the number of correct options marked.

76. Let $f(x)=\begin{cases}\int_0^x |1-t|\,dt, & x>1\\ x-\frac{1}{2}, & x\le 1\end{cases}$. Then

(a) $f(x)$ is continuous at $x = 1$

(b) $f(x)$ is not continuous at $x = 1$

(c) $f(x)$ is differentiable at $x = 1$

(d) $f(x)$ is not differentiable at $x = 1$

77. The angle of intersection between the curves $y = [|\sin x| + |\cos x|]$ and $x^2 + y^2 = 10$, where $[x]$ denotes the greatest integer $\le x$, is

(a) $\tan^{-1}(3)$ (b) $\tan^{-1}(-3)$

(c) $\tan^{-1}(\sqrt{3})$ (d) $\tan^{-1}(1/\sqrt{3})$

78. If $u(x)$ and $v(x)$ are two independent solutions of the differential equation $\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$, then additional solution(s) of the given differential equation is(are)

(a) $y = 5\,u(x) + 8\,v(x)$

(b) $y = c_1\{u(x) - v(x)\} + c_2\,v(x)$, c_1 and c_2 are arbitrary constants

(c) $y = c_1\,u(x)\,v(x) + c_2\,u(x)/v(x)$, c_1 and c_2 are arbitrary constants

(d) $y = u(x)\,v(x)$

79. For the two events A and B, let $P(A) = 0.7$ and $P(B) = 0.6$. The necessarily false statement(s) is/are

(a) $P(A \cap B) = 0.35$ (b) $P(A \cap B) = 0.45$

(c) $P(A \cap B) = 0.65$ (d) $P(A \cap B) = 0.28$

80. If the circle $x^2+y^2+2gx+2fy+c=0$ cuts the three circles $x^2+y^2-5=0$, $x^2+y^2-8x-6y+10=0$ and $x^2 + y^2 - 4x + 2y - 2 = 0$ at the extremities of their diameters, then

(a) $c = -5$ (b) $fg=\frac{147}{25}$

(c) $g + 2f = c + 2$ (d) $4f = 3g$

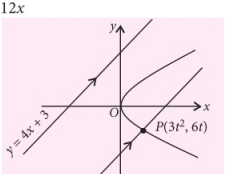
SOLUTIONS

1. (c): $\because$ Foci are $\left(\pm\sqrt{a^2-b^2},0\right)$ *i.e.* $\left(\pm\sqrt{119},0\right)$

$\therefore$ Radius = distance between $\left(0,\sqrt{2}\right)$ and $\left(\pm\sqrt{119},0\right)=\sqrt{119+2}=11$

2. (b) : $y = 4x + 3$ ...(1)

$y^2 = 12x$ ...(2)



Let $P(3t^2, 6t)$ be the foot of the normal which is parallel to (1)

From (2), $\left[\frac{dy}{dx}\right]_{\text{at } P}=-t$

$\therefore$ $-t = 4$ (slope of (1)] $\Rightarrow t = -4 \Rightarrow P(48, -24)$

$\therefore$ Normal at P is $y + 24 = 4(x - 48)$

$y = 4x - 216$ ...(3)

$\therefore$ Distance between parallel lines (1) and (3)

$=\frac{3-(-216)}{\sqrt{4^2+(-1)^2}}=\frac{219}{\sqrt{17}}$

3. (a) : $\because$ Product of the roots

$$= \frac{pq}{pq} = 1 = \tan A \tan B$$

$\therefore \quad A + B = \dfrac{\pi}{2} \quad \Rightarrow \quad \Delta$ is right angled.

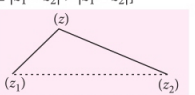
4. (b) : Number of possible relations from A to $B = 2^{pq}$

5. (a) : $\because \quad \left[x - \dfrac{\pi}{2}\right]$ is always an integer and $\tan k\pi$ $(k \in I)$ is always zero.

$\therefore \quad f(x) = 0 \ \forall \ x \in R$

$\therefore$ $f(x)$ is continuous for all values of x.

6. (a) : If $|z - z_1| + |z - z_2| = K$ and $K > |z_1 - z_2|$ then locus of z is an ellipse [Here, $K = 2\,|z_1 - z_2| > |z_1 - z_2|$]



7. (b) :

$\because \ {}^nC_0 + {}^nC_1 x + {}^nC_2 \cdot x^2 + ... + {}^nC_n \cdot x^n = (1+x)^n$

On integrating both sides between the limits 0 to 2,

$$\int_0^2 ({}^nC_0 + {}^nC_1 \cdot x + {}^nC_2 \cdot x^2 + ... + {}^nC_n x^n)dx = \int_0^2 (1+x)^n \, dx$$

$$\Rightarrow \left[{}^nC_0 x + {}^nC_1 \cdot \frac{x^2}{2} + {}^nC_2 \cdot \frac{x^3}{3} + ... + {}^nC_n \cdot \frac{x^{n+1}}{n+1}\right]_0^2 = \left[\frac{(1+x)^{n+1}}{n+1}\right]_0^2$$

$$\Rightarrow \left[\frac{2}{1} \cdot {}^nC_0 + \frac{2^2}{2} \cdot {}^nC_1 + \frac{2^3}{3} \cdot {}^nC_2 + ... + \frac{2^{n+1}}{n+1} \cdot {}^nC_n\right] = \frac{3^{n+1} - 1}{n+1}$$

8. (c): 3 consonants out of 7 and 2 vowels out of 4 can be selected in ${}^7C_3 \times {}^4C_2$ ways

$\therefore$ The number of words that can be made

$= {}^7C_3 \times {}^4C_2 \times \underline{5} = 25200$

9. (a) : $1! + 2! + 3! + 4! + ... + 11!$

$= 1 + 2 + 6 + 24 + 120 + ... = 9 + 12K$

[$\because$ After 1st 3 terms all are divisible by 12]

$\therefore$ Remainder = 9

10. (c): $\because$ t_r of 1, 8, 21, 40, 65, ... is given by $t_r = 1 + 7 + 13 + 19 + 25 + ... $ to r terms

$$= \frac{r}{2}\{2 \times 1 + (r-1)6\} = r(3r-2)$$

$$\therefore \ t_r \text{ of given series} = \frac{r(3r-2)}{\underline{r}} = \frac{3r-2}{\underline{r-1}} = \frac{3(r-1)+1}{\underline{r-1}}$$

$$= \frac{3}{\underline{r-2}} + \frac{1}{\underline{r-1}}$$

$\therefore$ Sum of given series $= 3e + e = 4e$

$\because \quad 2 < e < 3 \quad \therefore \quad 8 < 4e < 12 \quad \Rightarrow \quad 8 < S < 12$

11. (b) :

$$\because \ f(x) = \frac{2-1}{\underline{1}}x + \frac{2^2-1}{\underline{2}}x^2 + \frac{2^3-1}{\underline{3}}x^3 + ...$$

$$= \left(\frac{2x}{\underline{1}} + \frac{(2x)^2}{\underline{2}} + \frac{(2x)^3}{\underline{3}} + ...\right) - \left(\frac{x}{\underline{1}} + \frac{x^2}{\underline{2}} + \frac{x^3}{\underline{3}} + ...\right)$$

$$= \left[1 + \frac{2x}{\underline{1}} + \frac{(2x)^2}{\underline{2}} + \frac{(2x)^3}{\underline{3}} + ...\right] - \left[1 + \frac{x}{\underline{1}} + \frac{x^2}{\underline{2}} + ...\right]$$

$$= e^{2x} - e^x$$

$\therefore \quad f(x) = 0 \quad \Rightarrow \quad e^x(e^x - 1) = 0$

$\Rightarrow \quad e^x - 1 = 0 \qquad [\because \ e^x \neq 0]$

$\Rightarrow \quad x = 0$ is the only real solution.

12. (b) :

$$\because \ \frac{2}{(1-x)(2-x)} = 2\left(\frac{1}{1-x} - \frac{1}{2-x}\right)$$

$$= 2(1-x)^{-1} - \left(1 - \frac{x}{2}\right)^{-1}$$

$$= 2[1 + x + x^2 + x^3 + ... \text{ to } \infty] - \left[1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^3 + ... \text{ to } \infty\right]$$

[$\because \ |x| < 1$]

$\therefore$ Required coefficient of $x^3 = 2 - \dfrac{1}{8} = \dfrac{15}{8}$

13. (d) : $\because \quad \alpha + \beta = -p; \ \alpha\beta = q$

$\therefore \quad \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)$

$= -p^3 - 3q(-p) = 3pq - p^3$

and, $\alpha^4 + \alpha^2\beta^2 + \beta^4 = (\alpha^2 + \beta^2)^2 - (\alpha\beta)^2$

$= \{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - q^2$

$= (p^2 - 2q)^2 - q^2 = (p^2 - q)(p^2 - 3q)$

14. (c): $\because$ Each number (from 1 to 6) can be arranged in 12 throws

$$= \frac{|12}{|2\ |2\ |2\ |2\ |2\ |2} = \frac{|12}{2^6}$$

$$\therefore \quad \text{Required probability} = \frac{|12/2^6}{\underbrace{6.6.6.6. \ldots\ldots 6}_{12 \text{ times}}} = \frac{|12}{2^6 \cdot 6^{12}}$$

15. (c): $\because \quad f'(x) \le 5 \quad \therefore \int_2^7 f'(x)dx \le \int_2^7 5dx$

$$\Rightarrow \quad [f(x)]_2^7 \le [5x]_2^7 \quad \Rightarrow \quad f(7) - f(2) \le 25$$

$$\Rightarrow \quad f(7) \le 28 \quad [\because \ f(2) = 3]$$

$\therefore$ Maximum possible value of $f(x)$ at $x = 7$ is 28.

16. (a) : Let $\frac{\pi}{5} = \theta$

$$\therefore \quad 4\cot 4\theta = \frac{4}{\tan 4\theta} = \frac{4(1 - \tan^2 2\theta)}{2\tan 2\theta}$$

$$= 2\cot 2\theta - 2\tan 2\theta = \frac{2(1 - \tan^2\theta)}{2\tan\theta} - 2\tan 2\theta$$

$= \cot\theta - \tan\theta - 2\tan 2\theta$

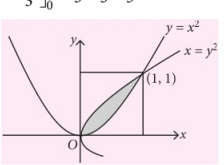
$$\therefore \quad \tan\theta + 2\tan 2\theta + 4\cot 4\theta = \cot\theta = \cot\frac{\pi}{5}$$

17. (b) : $1 \le 3x^2 + 1 \le 6 \quad \Rightarrow \quad x^2 \le \frac{5}{3}$

$$\Rightarrow \quad x \in \left[-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right]$$

18. (a) : Required area $= \int_0^1 (\sqrt{x} - x^2)dx$

$$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$



19. (a) : Let the required point on $y^2 = 64x$ ...(1) be $P(16t^2, 32t)$

$$\left[\frac{dy}{dx}\right]_{\text{at } P} = \frac{1}{t} = \text{slope of tangent at } P$$

If P is nearest to $4x + 3y + 35 = 0$ ...(2)

then tangent to (1) at P will be parallel to (2)

$$\therefore \quad \frac{1}{t} = -\frac{4}{3} \quad \left[\because \text{ slope of (2) is } -\frac{4}{3}\right]$$

$$\Rightarrow \quad t = -\frac{3}{4} \quad \Rightarrow \quad P(9, -24)$$

20. (a): $\because \quad y = mx + \frac{a}{m}$ is always a tangent to $y^2 = 4ax$

$$\therefore \quad y = mx + \frac{2\sqrt{3}}{m} \quad \text{...(1) is a tangent to}$$

$y^2 = 8\sqrt{3}x$ ...(2)

On solving (1) with $4x^2 - y^2 = 4$ ...(3), we get

$$4x^2 - \left(mx + \frac{2\sqrt{3}}{m}\right)^2 = 4$$

$$\Rightarrow \quad (4 - m^2)x^2 - 4\sqrt{3}x - \left(\frac{12}{m^2} + 4\right) = 0$$

Its roots must be equal for common tangent

$$\therefore \quad \text{We have } 48 = 4(4 - m^2)\left\{-\left(\frac{12}{m^2} + 4\right)\right\}$$

$$\Rightarrow \quad (m^2 - 6)(m^2 + 2) = 0 \quad \text{(on simplifying)}$$

$$\Rightarrow \quad m = \pm\sqrt{6} \quad [\because \ m^2 + 2 \ne 0]$$

For positive slope, $m = \sqrt{6}$

$\therefore$ Required tangent is $y = \sqrt{6}x + \sqrt{2}$

21. (d) : $\because \quad \alpha$ is a root of $x^2 + 3p^2x + 5q^2 = 0$

$$\therefore \quad \alpha^2 + 3p^2\alpha + 5q^2 = 0 \quad \text{...(1)}$$

$\because \quad \beta$ is a root of $x^2 + 9p^2x + 15q^2 = 0$

$$\therefore \quad \beta^2 + 9p^2\beta + 15q^2 = 0 \quad \text{...(2)}$$

Now, let $f(x) = x^2 + 6p^2x + 10q^2$

$$\therefore \quad f(\alpha) = \alpha^2 + 6p^2\alpha + 10q^2 = (\alpha^2 + 3p^2\alpha + 5q^2) + (3p^2\alpha + 5q^2)$$

$= 0 + 3p^2\alpha + 5q^2$ [from (1)]

$= 3p^2\alpha + 5q^2 > 0 \quad [\because \ \alpha > 0]$

$f(\beta) = \beta^2 + 6p^2\beta + 10q^2 = (\beta^2 + 9p^2\beta + 15q^2) - (3p^2\beta + 5q^2)$

$= 0 - (3p^2\beta + 5q^2) < 0 \quad [\because \ \beta > 0]$

$\therefore \quad f(\alpha)$ and $f(\beta)$ are of opposite sign

$\therefore \quad f(x) = 0$ will have a real root γ such that $\alpha < \gamma < \beta$

22. (d) : $\because\ (1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots\ldots + {}^nC_nx^n \quad \ldots(1)$

$(x+1)^n = {}^nC_0x^n + {}^nC_1x^{n-1} + {}^nC_2x^{n-2} + \ldots + {}^nC_n \quad \ldots(2)$

On multiplying and equating the coefficient of x^n from both sides

$$\therefore\ ({}^nC_1)^2 + ({}^nC_2)^2 + ({}^nC_3)^2 + \ldots\ldots + ({}^nC_n)^2 = {}^{2n}C_n - 1 \quad [\because\ {}^nC_0 = 1]$$

23. (c): All possibilities are {BB, BG, GB, GG}
When 1 of them is known as a boy then sample space will be {BB, BG, GB} *i.e.* 3.
Now, these will be 2 favourable cases {BG, GB} in which other child will be a girl.

$$\therefore\ \text{Required probability} = \frac{2}{3}$$

24. (a) : Let $\frac{2\pi}{n} = \theta$

$$\therefore\ A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore\ A^2 = \begin{pmatrix} \cos 2\theta & \sin 2\theta & 0 \\ -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ etc}$$

$$\therefore\ A^n = \begin{pmatrix} \cos n\theta & \sin n\theta & 0 \\ -\sin n\theta & \cos n\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos 2\pi & \sin 2\pi & 0 \\ -\sin 2\pi & \cos 2\pi & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

Also, $A^{n-1} \neq I$

25. (b) : As 3 columns can be arranged in $\lfloor 3 = 6$ ways, these will be six distinct choices for P and in each case $\det(P) = +1$ or -1.

26. (c): $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{n!\,\pi}{720}\right) = \sin\frac{\pi}{720} + \sin\frac{\pi}{360} + \sin\frac{\pi}{120} + \sin\frac{\pi}{30} + \sin\frac{\pi}{6} + (\sin\pi + \sin 7\pi + \ldots\ldots \text{ to } \infty)$

$$= \sin\frac{\pi}{6} + \sin\frac{\pi}{30} + \sin\frac{\pi}{120} + \sin\frac{\pi}{360} + \sin\frac{\pi}{720}$$

$[\because$ All terms within bracket are zero$]$

27. (a) : $\alpha + \beta = 1,\ \alpha\beta = -1$

Now, $S_{n+1} = \alpha^{n+1} + \beta^{n+1}$

$= (\alpha^n + \beta^n)(\alpha + \beta) - (\alpha^n\beta + \alpha\beta^n)$

$= S_n(1) - \alpha\beta(\alpha^{n-1} + \beta^{n-1})$

$= S_n - (-1)\ S_{n-1} = S_n + S_{n-1}$

28. (a) : $\Sigma a^2\ 2R \sin A \sin(B - C)$

$= \Sigma a^2\ 2R \sin(\pi - \overline{B + C}) \sin(B - C)$

$= 2R\ \Sigma a^2 \sin(B + C) \sin(B - C)$

$= 2R\ \Sigma a^2 (\sin^2 B - \sin^2 C)$

$$= 2R\,\Sigma a^2 \left\{\left(\frac{b}{2R}\right)^2 - \left(\frac{c}{2R}\right)^2\right\} = \frac{1}{2R}\,\Sigma a^2(b^2 - c^2) = 0$$

29. (b) : $1 + z + z^3 + z^4 = 0 \Rightarrow (z^3 + 1)(z + 1) = 0$

$\Rightarrow (z + 1)^2(z^2 - z + 1) = 0 \Rightarrow z = -1, -1, -\omega, -\omega^2$

Distinct roots are (-1), $(-\omega)$ and $(-\omega^2)$

Corresponding points are

$(-1, 0)$, $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ and they represent vertices of an equilateral triangle.

30. (c): Let $x = 20^{301}$

$$\therefore\ \log_{10} x = 301 \log_{10} 20 = 301\left(1 + \log_{10} 2\right)$$

$= 301\ (1.301) = 391.601$

$\therefore$ The number of digits in $20^{301} = 392$

31. (d) : $\because\ \sqrt{y} = \cos^{-1} x$

$$\therefore\ y = (\cos^{-1} x)^2 \ \Rightarrow\ y_1 = 2(\cos^{-1} x)\left\{-\frac{1}{\sqrt{1 - x^2}}\right\}$$

$$\Rightarrow\ (1 - x^2)y_1^2 = 4(\cos^{-1} x)^2 = 4y$$

$$\Rightarrow\ (1 - x^2)\cdot 2y_1y_2 - 2xy_1^2 = 4y_1$$

$$\Rightarrow\ (1 - x^2)y_2 - xy_1 = 2 \ \Rightarrow\ c = 2$$

32. (c): $\displaystyle\frac{dy}{dx} + \frac{1}{1 + x^2}\cdot y = \frac{e^{\tan^{-1} x}}{1 + x^2}$

Here, $\displaystyle P = \frac{1}{1 + x^2};\ Q = \frac{e^{\tan^{-1} x}}{1 + x^2}$

$$\therefore\ \text{I.F} = e^{\int P\cdot dx} = e^{\int \frac{dx}{1 + x^2}} = e^{\tan^{-1} x}$$

33. (c): $\log_{101}\log_7(\sqrt{x+7}+\sqrt{x})=0$

$\Rightarrow \log_7(\sqrt{x+7}+\sqrt{x})=1$

$\Rightarrow \sqrt{x+7}+\sqrt{x}=7$, which is clearly satisfied by $x=9$

34. (a) : $\because$ Difference of roots in both are same

$\therefore \alpha-\beta=(\alpha+h)-(\beta+h)$

$\Rightarrow (\alpha-\beta)^2=\{(\alpha+h)-(\beta+h)\}^2$

$\Rightarrow (\alpha+\beta)^2-4\alpha\beta$

$=\{(\alpha+h)+(\beta+h)\}^2-4(\alpha+h)(\beta+h)$

$\Rightarrow \left(-\frac{b}{a}\right)^2-4\left(\frac{c}{a}\right)=\left(-\frac{q}{p}\right)^2-4\cdot\frac{r}{p}$

$\Rightarrow \frac{b^2-4ac}{q^2-4pr}=\frac{a^2}{p^2}$

$\Rightarrow D_1:D_2=a^2:p^2$

[Note : Question should be ratio of their discriminants]

35. (c): $a(x+1)(x-2)+b(x-2)(x-1)+c(x-1)(x+1)=2x^2+5x+1$

On equating, $a+b+c=2$ ...(1)

$-a-3b+0\cdot c=5$ ...(2)

$-2a+2b-c=1$ ...(3)

$\therefore D=\begin{vmatrix}1&1&1\\-1&-3&0\\-2&2&-1\end{vmatrix}$

$=1(3-0)-(1-0)+(-2-6)=-6\neq 0$

$\therefore$ A unique solution exists for above three equations

$\therefore$ exactly one choice for each of a, b, c exists.

36. (a) : $\because$ $f(x), f(2x), f(4x)$ are in H.P.

$\therefore f(2x)=\frac{2f(x)f(4x)}{f(x)+f(4x)}$

$\Rightarrow 2x+\frac{1}{2}=\frac{2\left(x+\frac{1}{2}\right)\left(4x+\frac{1}{2}\right)}{\left(x+\frac{1}{2}\right)+\left(4x+\frac{1}{2}\right)}$

$\Rightarrow 4x+1=\frac{(2x+1)(8x+1)}{5x+1}\Rightarrow 4x^2-x=0$

$\Rightarrow x=0,\frac{1}{4}$

But, for $x=0$, $f(x), f(2x), f(4x)$ will be same

$\therefore x=\frac{1}{4}$ is the only real value of x.

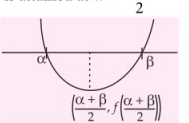
37. (d) : $\because f(x)=x^2+bx+c$ (b, c are constants) in which coefficient of x^2 is positive (= 1)

$\therefore$ Graph of $f(x)$ will be a parabola and concave upward.

$\therefore$ $f(x)$ is neither one-to-one nor onto.

38. (a) : $\because f(x)=x^2+bx+c=0$ has 2 distinct real roots α, β

$\therefore$ Minima is attained at $x=\frac{\alpha+\beta}{2}$



$\therefore$ Tangent $\left(\frac{\alpha+\beta}{2}, f\left(\frac{\alpha+\beta}{2}\right)\right)$ will be parallel to x-axis and so it will make 0° with positive direction of x-axis.

39. (c): Putting $y=vx$ so that $\frac{dy}{dx}=v+x\cdot\frac{dv}{dx}$

$\therefore$ Given equation reduces to

$v\cdot\left(v+x\frac{dv}{dx}\right)=v^2+\frac{\phi(v^2)}{\phi'(v^2)}\Rightarrow \frac{v\cdot\phi'(v^2)dv}{\phi(v^2)}=\frac{dx}{x}$

On integrating, $\frac{1}{2}\log\phi(v^2)=\log x+\frac{1}{2}\log c$

$\Rightarrow \phi(v^2)=cx^2\Rightarrow \phi\left(\frac{y^2}{x^2}\right)=cx^2$

40. (d) : $\lim_{x\to 2}\frac{f(4)-f(x^2)}{x-2}\left[\frac{0}{0}\right]$

$=\lim_{x\to 2}\frac{-f'(x^2)\cdot 2x}{1}=-4f'(4)=-20$

41. (a) : $\lim_{x\to 0}\frac{-\int_0^{x^2}\cos t^2\,dt}{x\sin x}\left[\frac{0}{0}\right]$

$=\lim_{x\to 0}\frac{\cos(x^2)^2\cdot 2x}{\sin x+x\cos x}=\lim_{x\to 0}\frac{2\cos x^4}{\frac{\sin x}{x}+\cos x}=\frac{2}{1+1}=1$

42. (c): $\because y=3\sin\sqrt{\frac{\pi^2}{16}-x^2}$

is defined when $\frac{\pi^2}{16} - x^2 \geq 0$

i.e. when $\left(x + \frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right) \leq 0$

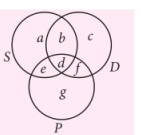
i.e. when $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

when $x = 0,\ y = 3\sin\frac{\pi}{4} = \frac{3}{\sqrt{2}}$

when $x = \pm\frac{\pi}{4},\ y = 0$

$\therefore\ 0 \leq y \leq \frac{3}{\sqrt{2}}$

43. (a) :



$a + b + c + d + e + f + g = 265$

$a + b + d + e = 200$

$b + c + d + f = 110$

$d + e + f + g = 55$

$b + d = 60$

$d + e = 30$

$d = 10$

On solving

a	b	c	d	e	f	g
120	50	40	10	20	10	15

$\therefore$ Number of persons who like only dancing and painting $= f = 10$

44. (a) : $y = (\cos x + y)^{1/2} \Rightarrow y^2 = \cos x + y$

$\Rightarrow\ 2y\cdot\frac{dy}{dx} = -\sin x + \frac{dy}{dx} \Rightarrow (2y-1)\frac{dy}{dx} = -\sin x$

$\Rightarrow\ (2y-1)\frac{d^2y}{dx^2} + 2\frac{dy}{dx}\cdot\frac{dy}{dx} = -\cos x$

45. (a) : If z_1, z_2, z_3 be vertices of an equilateral triangle, then

$$\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$$

Now, $\sum\frac{1}{(\alpha z_1 + \beta) - (\alpha z_2 + \beta)} = \frac{1}{\alpha}\sum\frac{1}{z_1 - z_2} = 0$

$\therefore\ \alpha z_1 + \beta, \alpha z_2 + \beta, \alpha z_3 + \beta$ will be vertices of an equilateral triangle.

46. (b) :

$$\lim_{x\to 0}\frac{2a\sin x - \sin 2x}{x^3}\cdot\frac{1}{\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^3}\left(\lim_{x\to 0}\cos^3 x\right)$$

$$\lim_{x\to 0}\frac{2a\sin x - \sin 2x}{x^3}\left[\frac{0}{0}\right] = \lim_{x\to 0}\frac{2a\cos x - 2\cos 2x}{3x^2},$$

which will exist only if it is in $\frac{0}{0}$ form

$\therefore$ we must have, $2a\cos 0 - 2\cos 0 = 0 \Rightarrow a = 1$

Further, if $a = 1$, Given limit becomes

$$\lim_{x\to 0}\frac{2\sin x - \sin 2x}{\tan^3 x} = \lim_{x\to 0}\frac{2\sin x(1-\cos x)}{\sin^3 x}\cdot\cos^3 x$$

$$= 2\cdot\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)\cdot\frac{1}{\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^2}\cdot\lim_{x\to 0}\cos^3 x = 1$$

47. (a) : $\int_0^2 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx$

$$= \int_0^1 (2x^2 + 1)dx + \int_1^2 (4x^3 - 1)dx$$

$$= \left[\frac{2x^3}{3} + x\right]_0^1 + \left[x^4 - x\right]_1^2 = \left(\frac{2}{3} + 1\right) + (14 - 0) = \frac{47}{3}$$

48. (c): $|z|^2 + |z-3|^2 + |z-i|^2$. Let $z = x + iy$

$= (x^2 + y^2) + \{(x-3)^2 + y^2\} + \{x^2 + (y-1)^2\}$

$= 3x^2 + 3y^2 - 6x - 2y + 10$

$$= 3\left\{x^2 + y^2 - 2x - \frac{2y}{3} + \frac{10}{3}\right\}$$

$$= 3\left\{(x-1)^2 + \left(y - \frac{1}{3}\right)^2\right\} + \frac{20}{3}$$

$$= 3\left|(x-1) + i\left(y - \frac{1}{3}\right)\right|^2 + \frac{20}{3} = 3\left|z - \left(1 + \frac{i}{3}\right)\right|^2 + \frac{20}{3}$$

$\therefore$ Minimum value is obtained when $z = 1 + \frac{i}{3}$

49. (b) : $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ ... (1)

On squaring, $(x+1) + (x-1) - 2\sqrt{x^2-1} = 4x - 1$

$2x - 1 = -2\sqrt{x^2 - 1}$ ... (2)

On squaring again, $4x^2 - 4x + 1 = 4(x^2 - 1)$

$4x = 5 \Rightarrow x = \frac{5}{4}$

However, $x = \frac{5}{4}$ does not satisfy (2) as LHS is positive while RHS is negative.
Hence, no solution exists.

50. (b) : $(7x+5)^2 + (7y+3)^2 = \lambda^2(4x+3y-24)^2$

$\Rightarrow \sqrt{\left(x+\frac{5}{7}\right)^2 + \left(y+\frac{3}{7}\right)^2} = \frac{|\lambda|}{7}\frac{|4x+3y-24|}{5} \cdot 5$

For parabola, $\frac{5|\lambda|}{7} = 1 \Rightarrow \lambda = \pm\frac{7}{5}$

51. (a) : $\sin^{-1}\frac{x}{13} + \operatorname{cosec}^{-1}\frac{13}{12} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{x}{13} + \cos^{-1}\frac{5}{13} = \frac{\pi}{2}$

$\Rightarrow \frac{x}{13} = \frac{5}{13} \quad \left[\because \sin^{-1}t + \cos^{-1}t = \frac{\pi}{2}\right]$

$\therefore \quad x = 5$

52. (a) : $x + y = 0,\ 5x + y = 4,\ x + 5y = 4$

On solving, we get vertices as

$(1,1), \left(\frac{2}{3}, \frac{2}{3}\right) \& (-1, 1)$

Clearly, they form an isosceles triangle

53. (a) : $\because e^{x^4}$ is increasing function and always +ve

$\therefore \quad \int_0^2 e^{x^4}(x-\alpha)\,dx = 0$ shows that $x - \alpha$ must be positive as well as negative in $(0, 2) \Rightarrow \alpha \in (0, 2)$

54. (a) :

$\because \quad T_{r+1}$ in $\left(ax^2 + \frac{1}{bx}\right)^{13} = {}^{13}C_r (ax^2)^{13-r}\left(\frac{1}{bx}\right)^r$

$= {}^{13}C_r \cdot \frac{a^{13-r}}{b^r} \cdot x^{26-3r}$

For coefficient of x^8, we have $26 - 3r = 8 \Rightarrow r = 6$

$\Rightarrow$ coefficient of $x^8 = {}^{13}C_6 \frac{a^7}{b^6}$

$\because \quad T_{r+1}$ in $\left(ax - \frac{1}{bx^2}\right)^{13} = {}^{13}C_r (ax)^{13-r}\left(-\frac{1}{bx^2}\right)^r$

For coeff. of x^{-8}, we have $13 - 3r = -8 \Rightarrow r = 7$

$\therefore \quad$ coeff. of $x^{-8} = -{}^{13}C_7 \frac{a^6}{b^7}$

Now, ${}^{13}C_6 \frac{a^7}{b^6} = -{}^{13}C_7 \frac{a^6}{b^7} \Rightarrow ab = -1 \Rightarrow ab + 1 = 0$

55. (c): $f(x) = \begin{cases} a\sin x + be^x & , x \geq 0 \\ -a\sin x + be^{-x}, & x < 0 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} a\cos x + be^x & , x \geq 0 \\ -a\cos x - be^{-x}, & x < 0 \end{cases}$

$\because \quad f(x)$ is differentiable at $x = 0$
$\Rightarrow$ L.H.D. = R.H.D. at $x = 0$
$\Rightarrow \quad a + b = -a - b \Rightarrow a + b = 0$

56. (c): On putting $x = 1$, we get
L.H.S. $= \log_e a - 2\log_e b + \log_e c$

$= \log_e \frac{ac}{b^2} = \log_e 1 = 0 \quad \left[\because b^2 = ac\right]$

= R.H.S.
$\therefore \quad 1$ is a root of given equation.

$\therefore$ other root $= \frac{\log_e c}{\log_e a} = \log_a c$

57. (d) : $f(0) = 0, f(1) = \sin 1 \quad \therefore \ f(0) \neq f(1)$
$\Rightarrow$ 3rd condition of Rolle's theorem is violated in [0, 1]
Also, $f(x)$ is not differentiable at $x = 0$
$\therefore$ 2nd condition of Rolle's theorem as well as LMVT are violated in $[-1, 1]$ as $0 \in (-1, 1)$
f satisfies the condition LMVT on [0, 1]

58. (b) : Let $z_1 = re^{i\theta}$
As z_1, z_2, z_3 are vertices of an equilateral triangle inscribed on $|z| = 1$ and taken in counter-clockwise direction, z_2 and z_3 may be taken as $z_2 = re^{i\left(\theta + \frac{2\pi}{3}\right)}$ and $z_3 = re^{i\left(\theta + \frac{4\pi}{3}\right)}$

$\therefore \quad z_1 z_2 z_3 = r^3 \cdot e^{i(3\theta + 2\pi)}$
$= r^3[\cos(2\pi + 3\theta) + i\sin(2\pi + 3\theta)]$
$= r^3(\cos 3\theta + i\sin 3\theta) = r^3 e^{i\cdot 3\theta} = z_1^3$

59. (c):

$\because \quad g'(0) = \lim_{h\to 0}\frac{g(0+h) - g(0)}{h} = \lim_{h\to 0}\frac{g(h) - 0 \cdot f'(0)}{h}$

$= \lim_{h\to 0}\frac{hf'(h) - 0(1)}{h} \quad [\because f'(0) = 1]$

$= \lim_{h\to 0} f'(h) = f'(0) = 1$

60. (c): For any real number x, we know that
$x - 1 < [x] \le x$

$\therefore \quad n\sqrt{2} - 1 < [n\sqrt{2}] \le n\sqrt{2}$

$\Rightarrow \quad \sqrt{2} - \frac{1}{n} < \frac{[n\sqrt{2}]}{n} \le \sqrt{2}$

$\Rightarrow \quad \lim_{n\to\infty}\left(\sqrt{2} - \frac{1}{n}\right) < \lim_{n\to\infty}\frac{[n\sqrt{2}]}{n} \le \lim_{n\to\infty}\sqrt{2}$

$\Rightarrow \quad \sqrt{2} < \lim_{n\to\infty}\frac{[n\sqrt{2}]}{n} \le \sqrt{2} \Rightarrow \lim_{n\to\infty}\frac{[n\sqrt{2}]}{n} = \sqrt{2}$

61. (d) : (i) $\because IAI^{-1} = (AI)I^{-1} = A(II^{-1}) = AI = A$
$\therefore \quad A = IAI^{-1}$, where I and I^{-1} are invertible matrices
$\Rightarrow$ Relation is reflexive
(ii) Let P and Q be invertible matrices
$\therefore \quad P^{-1}$ and Q^{-1} are also invertible
Now, if $B = PAQ^{-1}$ then $BQ = (PAQ^{-1})Q = PAI = PA$
$\Rightarrow \quad P^{-1}(BQ) = P^{-1}(PA) = IA = A$
$\Rightarrow \quad P^{-1}B(Q^{-1})^{-1} = A$
$\Rightarrow$ Relation is symmetric as P^{-1} and Q^{-1} both are invertible.
(iii) Let $A \sim B$ and $B \sim C$
i.e. $B = PAQ^{-1}$ and $C = PBQ^{-1}$
$\therefore \quad C = P(PAQ^{-1})Q^{-1}$
$= (PP)A(Q^{-1}Q^{-1}) = (PP)\,A(QQ)^{-1}$
$\Rightarrow \quad A \sim C \Rightarrow$ Relation is transitive also
Hence, relation is equivalence.

62. (a) : $\because$ A.M $\ge$ G.M

$\therefore \quad \frac{2^{\sin x} + 2^{\cos x}}{2} \ge \sqrt{2^{\sin x}\cdot 2^{\cos x}}$

$\because \quad -\sqrt{2} \le \sin x + \cos x \le \sqrt{2}$ ...(1))

$\Rightarrow \quad 2^{\sin x} + 2^{\cos x} \ge 2\cdot 2^{\frac{\sin x + \cos x}{2}} \ge 2\cdot 2^{\frac{-\sqrt{2}}{2}}$ (using (1))
$\ge 2^{1-\frac{1}{\sqrt{2}}}$

$\therefore \quad$ Minimum value of $2^{\sin x} + 2^{\cos x}$ is $2^{1-\frac{1}{\sqrt{2}}}$

63. (d) :

(i) $\because \quad \sec^2\theta - \tan^2\theta = 1 \quad$ or $\quad \sec^2\phi - \tan^2\phi = 1$
$\theta R\theta \quad$ or $\quad \phi R\phi \Rightarrow$ Reflexive
(ii) $\sec^2\theta - \tan^2\phi = 1$
$\Rightarrow \quad (1 + \tan^2\theta) - (\sec^2\phi - 1) = 1$
$\Rightarrow \quad \sec^2\phi - \tan^2\theta = 1$
$\therefore \quad \theta R\phi \Rightarrow \phi R\theta \Rightarrow$ symmetric
(iii) Let $\theta R\phi$ and $\phi R\Psi$ *i.e.* $\sec^2\theta - \tan^2\phi = 1$ ...(1)
and $\sec^2\phi - \tan^2\Psi = 1$ ...(2)
Adding (1) and (2),
$\sec^2\theta + (\sec^2\phi - \tan^2\phi) - \tan^2\Psi = 2$
$\Rightarrow \quad \sec^2\theta - \tan^2\Psi = 1 \Rightarrow \theta R\Psi \Rightarrow$ Transitive
Hence, relation is an equivalence relation.

64. (b) : As there is an acceleration, velocity at half time will be less than velocity at half distance
$\therefore \quad v_2 < v_1$

65. (b) : $t_n = \frac{1 + 9 + 11 + 13 + 15 + \ldots\ldots \text{ to } n \text{ terms}}{\lfloor n}$

$= \frac{1 + \frac{n-1}{2}\{2\times 9 + (n-1-1)2\}}{\lfloor n}$

$= \frac{1 + (n-1)(9 + n - 2)}{\lfloor n}$

$= \frac{n^2 + 6n - 6}{\lfloor n} = \frac{n}{\lfloor n-1} + \frac{6}{\lfloor n-1} - \frac{6}{\lfloor n}$

$= \frac{n-1+1}{\lfloor n-1} + \frac{6}{\lfloor n-1} - \frac{6}{\lfloor n} = \frac{1}{\lfloor n-2} + \frac{7}{\lfloor n-1} - \frac{6}{\lfloor n}$

$\therefore \quad \lim_{x\to\infty} t_n = 0$

66. (a) : $\because$ Cube roots of unity other than 1 are ω and ω^2. Let $\alpha = \omega^2$, $\beta = \omega$

$\therefore \quad S = \sum_{n=0}^{302}(-1)^n\left(\frac{\alpha}{\beta}\right)^n = \sum_{n=0}^{302}(-1)^n\,\omega^n$

$= 1 - \omega + \omega^2 - \omega^3 + \ldots\ldots + \omega^{302}$ (G.P.)

$= \frac{1-(-\omega)^{303}}{1-(-\omega)} = \frac{1+1}{-\omega^2} = -2\omega \quad [\because \ \omega^3 = 1]$

If $\alpha = \omega$, $\beta = \omega^2$ then $\frac{\alpha}{\beta} = \frac{\omega}{\omega^2} = \frac{\omega^4}{\omega^2} = \omega^2$

$S = \sum_{n=0}^{302}(-1)^n(\omega^2)^n = 1 - \omega^2 + \omega^4 - \ldots\ldots\ldots + \omega^{604}$

$= \frac{1-(-\omega^2)^{303}}{1-(-\omega^2)} = \frac{1+1}{-\omega} = -2\omega^2$

$\therefore \quad S = -2\omega$ or $-2\omega^2$

67. (a) : $\because \quad ae = 8; \quad \dfrac{2b^2}{a} = 24$ (given)

$\because \quad b^2 = a^2(e^2 - 1)$

$\Rightarrow \quad 12a = 64 - a^2 \Rightarrow a^2 + 12a - 64 = 0$

$\Rightarrow \quad (a + 16)(a - 4) = 0 \Rightarrow a = 4 \quad [\because \ a \neq -16]$

$\therefore \quad b^2 = 48$

$\therefore$ Required hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{48} = 1 \Rightarrow 3x^2 - y^2 = 48$$

68. (c): $\because \quad f(h) = f(0) + hf'(\theta h), \ [0 < \theta < 1] \quad ...(1)$

Let $f(x) = \cos x \ \therefore \ \cos h = \cos 0 + h(-\sin \theta h)$

$$\Rightarrow \quad \sin \theta h = \frac{1 - \cos h}{h} \Rightarrow \theta = \frac{1}{h} \sin^{-1}\left(\frac{1 - \cos h}{h}\right)$$

$$\Rightarrow \quad \lim_{h \to 0^+} \theta = \lim_{h \to 0^+} \frac{\sin^{-1}\left(\dfrac{1 - \cos h}{h}\right)}{h}$$

$$= \lim_{h \to 0^+} \frac{\sin^{-1}\left(\dfrac{1 - \cos h}{h}\right)}{\dfrac{1 - \cos h}{h}} \cdot \frac{(1 - \cos h)}{h^2} = 1 \cdot \frac{1}{2} = \frac{1}{2}$$

69. (a) : For $n = 1, |z|^2 \leq 1 \Rightarrow |z| \leq 1$

For $n = 2, \ |z|^2 \leq \dfrac{1}{2} \Rightarrow |z| \leq \dfrac{1}{\sqrt{2}}$ etc.

As $n \to \infty, \ |z| \leq \dfrac{1}{\sqrt{n}} \Rightarrow |z| \leq 0$, a point circle

$\therefore \ \bigcap_{n=1}^{\infty} X_n$ will be a point circle.

70. (d) : $M = \displaystyle\int_0^{\pi/2} \frac{\cos x \, dx}{x + 2}$

Putting $x = 2t \Rightarrow dx = 2dt$

$$M = \int_0^{\pi/4} \frac{2\cos 2t \, dt}{2t + 2} = \int_0^{\pi/4} \frac{\cos 2t \, dt}{t + 1}$$

$$= \left[\frac{1}{t+1}\left(\frac{\sin 2t}{2}\right)\right]_0^{\pi/4} - \int_0^{\pi/4} \left\{\frac{-1}{(t+1)^2}\right\} \cdot \left(\frac{\sin 2t}{2}\right) dt$$

$$= \frac{1}{\dfrac{\pi}{4} + 1} \cdot \frac{1}{2} - 0 + \int_0^{\pi/4} \frac{\sin t \cos t \, dt}{(t+1)^2}$$

$$= \frac{2}{\pi + 4} + N \quad \therefore \quad M - N = \frac{2}{\pi + 4}$$

71. (c):

$$\frac{1}{2\sin\dfrac{2\pi}{7}}\left[\sin\frac{4\pi}{7} + \left(\sin\frac{6\pi}{7} - \sin\frac{2\pi}{7}\right) + \left(\sin\frac{8\pi}{7} - \sin\frac{4\pi}{7}\right)\right]$$

$$= \frac{1}{2\sin\dfrac{2\pi}{7}}\left[\sin\left(\pi - \frac{\pi}{7}\right) - \sin\frac{2\pi}{7} + \sin\left(\pi + \frac{\pi}{7}\right)\right]$$

$$= \frac{1}{2\sin\dfrac{2\pi}{7}}\left[\sin\frac{\pi}{7} - \sin\frac{2\pi}{7} - \sin\frac{\pi}{7}\right] = -\frac{1}{2} < 0$$

72. (c): Let the following events :

E = He knows the correct answer

F = He randomly ticks

G = He answered correctly

$\therefore$ Required probability = $P(E / G)$

$$= \frac{P(G/E) \cdot P(E)}{P(G/E) \cdot P(E) + P(G/F) \cdot P(F)}$$

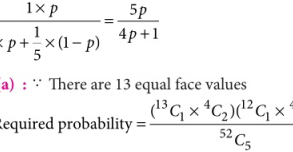
$$= \frac{1 \times p}{1 \times p + \dfrac{1}{5} \times (1 - p)} = \frac{5p}{4p + 1}$$

73. (a) : $\because$ There are 13 equal face values

$$\therefore \text{ Required probability} = \frac{({}^{13}C_1 \times {}^4C_2)({}^{12}C_1 \times {}^4C_3)}{{}^{52}C_5}$$

$$= \frac{13 \cdot 6 \cdot 12 \cdot 4 (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} = \frac{6}{4165}$$

74. (c): Graph of $f(x)$ is as shown in figure.



$$\therefore \quad \text{Reqd. area} = 3 \times \left(\frac{1}{2} \cdot 1 \cdot 1\right) + \frac{1}{2} \cdot 3 \cdot 6 = \frac{21}{2}$$

75. (a) : $\dfrac{dy}{dx} + \dfrac{y}{x \log_e x} = \dfrac{1}{x}$

$$\log_e x \cdot \frac{dy}{dx} + y \cdot \frac{1}{x} = \frac{\log_e x}{x} \Rightarrow \frac{d}{dx}(y \cdot \log_e x) = \frac{\log_e x}{x}$$

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- *Payment will be made after the MCQs are published.*
- *Kindly note that each question should be complete.*
- *Payment will be made only for complete questions.*
- *Preference will be given to the reader sending the maximum complete and correct questions. Other conditions apply. The decision of the Editor, MTG shall be final and binding.*

On integrating w.r.to x, $y\log_e x = \frac{(\log_e x)^2}{2} + C$

$\because$ when $x = e, y = 1 \Rightarrow 1 = \frac{1}{2} + C \therefore C = \frac{1}{2}$

$\therefore$ Required solution is $2y\log_e x = (\log_e x)^2 + 1$

$\Rightarrow 2y = \log_e x + \frac{1}{\log_e x}$

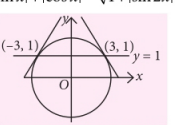
76. (a, d) :

$$\int_0^x |1-t|\,dt = \int_0^1 (1-t)dt + \int_1^x (t-1)dt = \frac{x^2}{2} - x + 1,$$

$$f(x) = \begin{cases} \frac{x^2}{2} - x + 1, x > 1 \\ x - \frac{1}{2}, x \le 1 \end{cases} \Rightarrow f'(x) = \begin{cases} x - 1, x > 1 \\ 1, x \le 1 \end{cases}$$

Clearly $f(x)$ is continuous but not differentiable at $x = 1$.

77. (a, b) : $|\sin x| + |\cos x| = \sqrt{1 + |\sin 2x|}$



So, $1 < |\sin x| + |\cos x| \le \sqrt{2}$. $y = [|\sin x| + |\cos x|] = 1$.

$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$

So, angle is either $\tan^{-1}(-3)$ or $\tan^{-1}(3)$.

78. (a, b) : Any linear combination of $u(x)$ and $v(x)$ will also be a solution.

79. (c, d) : $P(A \cup B) = 1.3 - P(A \cap B)$

Now $P(A) \le P(A \cup B) \le 1$

$\Rightarrow 0.7 \le 1.3 - P(A \cap B) \le 1$

$\Rightarrow 0.3 \le P(A \cap B) \le 0.6$.

80. (a, b, d) : Common chords of the circle will pass through the centres.

$c = -5, 8g + 6f = -35, 4g - 2f = -7$

So, $g = \frac{-14}{5}, f = \frac{-21}{10}$

MATHS MUSING

SOLUTION SET-137

1. (d): $3^{2014} = -(1-10)^{1007}$

$= -1 + 1007\cdot10 - 1007\cdot503\cdot10^2 + 1007\cdot503\cdot335\cdot10^3 - \ldots = \ldots. 2969$

2. (c): $x \to \pm\infty \Rightarrow y \to \infty$

$y'(x) = 0 \Rightarrow x = -r^{1/3}, \quad y(-r^{1/3}) > 0$

$s > r^{4/3} \Rightarrow s^3 > r^4$

3. (a): $i\omega^2 = z$, $(i^{101} + (i\omega^2)^{101})^{103}$

$= (i + i\omega)^{103} = -i(-\omega^2)^{103} = i\omega^2 = z$

4. (b): Last digit 2 or 4 and without 0, there are $4\cdot3\cdot2\cdot1\cdot2 = 48$ numbers. Last digit 0, without 3, there are $4\cdot3\cdot2\cdot1 = 24$ numbers. Last digit 2 or 4 and without 3, there are $3\cdot3\cdot2\cdot1\cdot2 = 36$ numbers.

Total = 48 + 24 + 36 = 108

There are $5\cdot5\cdot4\cdot3\cdot2 = 600$,

So probability $= \dfrac{108}{600} = \dfrac{9}{50}$

5. (b): $\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = 2\sqrt{2}\begin{bmatrix} \cos\frac{\pi}{12} & -\sin\frac{\pi}{12} \\ \sin\frac{\pi}{12} & \cos\frac{\pi}{12} \end{bmatrix}\begin{bmatrix} a_n \\ b_n \end{bmatrix}$

$= (2\sqrt{2})^n \begin{bmatrix} \cos\frac{\pi}{12} & -\sin\frac{\pi}{12} \\ \sin\frac{\pi}{12} & \cos\frac{\pi}{12} \end{bmatrix}^n \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$

$\begin{bmatrix} a_{22} \\ b_{22} \end{bmatrix} = (2\sqrt{2})^{21} \begin{bmatrix} \cos\frac{7\pi}{4} & -\sin\frac{7\pi}{4} \\ \sin\frac{7\pi}{4} & \cos\frac{7\pi}{4} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= 2^{30}\cdot2\sqrt{2}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\therefore \quad a_{22} = 2^{32}$

6. (c,d): $3 = \dfrac{2\sin(A+C)\sin C}{\cos A}$

$= 2(\tan A\cos C + \sin C)\sin C$

$= 2\sin^2 C(4\cos^2 C + 1) = 2s(5 - 4s),\ s = \sin^2 C$

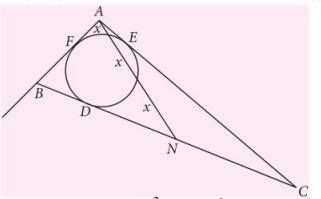
$8s^2 - 10s + 3 = 0,\ s = \dfrac{3}{4},\ \dfrac{1}{2}$

$C = \dfrac{\pi}{3},\ \dfrac{\pi}{4}$

7. (b): N is the midpoint of BC

$AE^2 = x\cdot 2x$

$2x^2 = (s-a)^2$ ...(i)



$2x^2 = DN^2 = \left(\dfrac{a}{2} - (s-b)\right)^2 = \dfrac{(b-c)^2}{4}$

$\therefore \quad (b + c - 10)^2 = (b-c)^2 \Rightarrow c = 5$

$AN^2 = \dfrac{b^2 - 25}{2} = 9x^2$

(i) $\Rightarrow (b-5)^2 = 8x^2$

$\therefore \quad b = 13,\ x = 2\sqrt{2},\ r = \sqrt{\dfrac{4\cdot1\cdot9}{14}} = \dfrac{6}{\sqrt{14}}$

8. (d): $AN = 3x = 6\sqrt{2}$

9. (0): The S.D. between the lines $\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{d} = \left|(\vec{c} - \vec{a})\cdot\hat{n}\right| = 0,\ \hat{n} = \dfrac{\vec{b}\times\vec{d}}{|\vec{b}\times\vec{d}|}$

$\vec{a} = 0,\ \vec{b} = \hat{i} - \hat{j} - \hat{k},\ \vec{c} = 3\hat{j},\ \vec{d} = \hat{i} - \hat{k}.$

10. (d):

P. $\int_0^1 \dfrac{dx}{\sqrt{2x - x^2}} = \dfrac{\pi}{2}$, **Q.** $\int_0^1 \sin\dfrac{\pi x}{2}\,dx = \dfrac{2}{\pi}$

R. $\int_0^1 \dfrac{dx}{\sqrt{4 - x^2}} = \dfrac{\pi}{6}$, **S.** $\int_0^1 \dfrac{dx}{\sqrt{2x + x^2}\,(1+x)} = \dfrac{\pi}{3}$

Solution Sender of Maths Musing

SET-137

1.	Sri Aditya, Vivek Senapathi, Shashank	: Hyderabad
2.	Gouri Sankar Adhikari	: Medinipur (WB)
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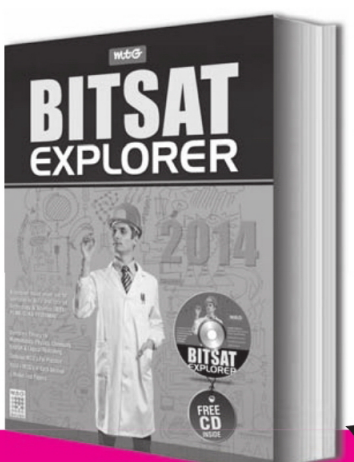

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SOLVED PAPER 2014

Karnataka CET

1. Which one of the following is not correct for the features of exponential function given by $f(x) = b^x$ where $b > 1$?

(a) For very large negative values of x, the function is very close to 0.

(b) The domain of the function is R, the set of real numbers.

(c) The point (1, 0) is always on the graph of the function.

(d) The range of the function is the set of all positive real numbers.

2. If $y = (1 + x)(1 + x^2)(1 + x^4)$, then $\frac{dy}{dx}$ at $x = 1$ is

(a) 20 (b) 28 (c) 1 (d) 0

3. If $y = (\tan^{-1}x)^2$, then $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1$ is equal to

(a) 4 (b) 0 (c) 2 (d) 1

4. If $f(x) = x^3$ and $g(x) = x^3 - 4x$ in $-2 \le x \le 2$, then consider the statements :

(i) $f(x)$ and $g(x)$ satisfy mean value theorem.

(ii) $f(x)$ and $g(x)$ both satisfy Rolle's theorem.

(iii) Only $g(x)$ satisfies Rolle's theorem.

Of these statements

(a) (i) and (ii) are correct

(b) (i) alone is correct

(c) None is correct

(d) (i) and (iii) are correct

5. Which of the following is not a correct statement?

(a) Mathematics is interesting.

(b) $\sqrt{3}$ is a prime.

(c) $\sqrt{2}$ is irrational.

(d) The sun is star.

6. If the function $f(x)$ satisfies $\lim_{x\to 1} \frac{f(x)-2}{x^2-1} = \pi$, then $\lim_{x\to 1} f(x) =$

(a) 1 (b) 2 (c) 0 (d) 3

7. The tangent to the curve $y = x^3 + 1$ at (1, 2) makes an angle θ with y- axis, then the value of $\tan\theta$ is

(a) $-\frac{1}{3}$ (b) 3 (c) –3 (d) $\frac{1}{3}$

8. If the function $f(x)$ defined by

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots\ldots + \frac{x^2}{2} + x + 1,$$

then $f'(0) =$

(a) $100 f'(0)$ (b) 100

(c) 1 (d) –1

9. If $f(x) = f(\pi + e - x)$ and $\int_e^{\pi} f(x)dx = \frac{2}{e+\pi}$, then $\int_e^{\pi} xf(x)dx$ is equal to

(a) $\pi - e$ (b) $\frac{\pi+e}{2}$ (c) 1 (d) $\frac{\pi-e}{2}$

10. If linear function $f(x)$ and $g(x)$ satisfy

$$\int[(3x-1)\cos x + (1-2x)\sin x]dx$$

$= f(x)\cos x + g(x)\sin x + C$, then

(a) $f(x) = 3(x - 1)$ (b) $f(x) = 3x - 5$

(c) $g(x) = 3(x - 1)$ (d) $g(x) = 3 + x$

11. The value of the integral $\int_{-\pi/4}^{\pi/4} \log(\sec\theta - \tan\theta)d\theta$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) π (d) $\frac{\pi}{2}$

12. $\int \frac{\sin 2x}{\sin^2 x + 2\cos^2 x} dx =$

(a) $-\log(1+\sin^2 x) + C$
(b) $\log(1+\cos^2 x) + C$
(c) $-\log(1+\cos^2 x) + C$
(d) $\log(1+\tan^2 x) + C$

13. Let S be the set of all real numbers. A relation R has been defined on S by $aRb \Leftrightarrow |a-b| \le 1$, then R is

(a) symmetric and transitive but not reflexive
(b) reflexive and transitive but not symmetric
(c) reflexive and symmetric but not transitive
(d) an equivalence relation

14. For any two real numbers, an operation $*$ defined by $a * b = 1 + ab$ is

(a) neither commutative nor associative
(b) commutative but not associative
(c) both commutative and associative
(d) associative but not commutative.

15. Let $f: N \to N$ defined by

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$ then f is

(a) onto but not one-one
(b) one-one and onto
(c) neither one-one nor onto
(d) one-one but not onto

16. Suppose $f(x) = (x+1)^2$ for $x \ge -1$. If $g(x)$ is a function whose graph is the reflection of the graph of $f(x)$ in the line $y = x$, then $g(x) =$

(a) $\frac{1}{(x+1)^2}; x > -1$ (b) $-\sqrt{x} - 1$

(c) $\sqrt{x} + 1$ (d) $\sqrt{x} - 1$

17. The domain of the function $f(x) = \sqrt{\cos x}$ is

(a) $\left[\frac{3\pi}{2}, 2\pi\right]$ (b) $\left[0, \frac{\pi}{2}\right]$

(c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

18. In a class of 60 students, 25 students play cricket and 20 students play tennis and 10 students play both the games, then the number of students who play neither is

(a) 45 (b) 0 (c) 25 (d) 35

19. Given $0 \le x \le \frac{1}{2}$ then the value of

$$\tan\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}} + \frac{\sqrt{1-x^2}}{\sqrt{2}}\right\} - \sin^{-1} x\right] \text{ is}$$

(a) 1 (b) $\sqrt{3}$ (c) -1 (d) $\frac{1}{\sqrt{3}}$

20. The value of $\sin(2\sin^{-1}0.8)$ is equal to

(a) 0.48 (b) sin 1.2°
(c) sin 1.6° (d) 0.96

21. If A is 3×4 matrix and B is a matrix such that $A'B$ and BA' are both defined, then B is of the type

(a) 4×4 (b) 3×4 (c) 4×3 (d) 3×3

22. The symmetric part of the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 6 & 8 & 2 \\ 2 & -2 & 7 \end{bmatrix} \text{ is}$$

(a) $\begin{bmatrix} 0 & -2 & -1 \\ -2 & 0 & -2 \\ -1 & -2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 8 & 0 \\ 3 & 0 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 2 \\ -1 & 2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 & 3 \\ 4 & 8 & 0 \\ 3 & 0 & 7 \end{bmatrix}$

23. If A is a matrix of order 3, such that $A\,(\text{adj } A) = 10\,I$, then $|\text{adj } A| =$

(a) 1 (b) 10 (c) 100 (d) $10\,I$

24. Consider the following statements :

(i) If any two rows or columns of a determinant are identical, then the value of the determinant is zero.
(ii) If the corresponding rows and columns of a determinant are interchanged, then the value of the determinant does not change.
(iii) If any two rows (or columns) of a determinant are interchanged, then the value of the determinant changes in sign.

Which of these are correct?

(a) (i) and (iii) (b) (i) and (ii)

(c) (i), (ii) and (iii) (d) (ii) and (iii)

25. The inverse of the matrix $A=\begin{bmatrix}2&0&0\\0&3&0\\0&0&4\end{bmatrix}$ is

(a) $\frac{1}{24}\begin{bmatrix}2&0&0\\0&3&0\\0&0&4\end{bmatrix}$ (b) $\begin{bmatrix}2&0&0\\0&3&0\\0&0&4\end{bmatrix}$

(c) $\frac{1}{24}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ (d) $\begin{bmatrix}\frac{1}{2}&0&0\\0&\frac{1}{3}&0\\0&0&\frac{1}{4}\end{bmatrix}$

26. If a, b and c are in A.P., then the value of $\begin{vmatrix}x+2&x+3&x+a\\x+4&x+5&x+b\\x+6&x+7&x+c\end{vmatrix}$ is

(a) 0 (b) $x-(a+b+c)$

(c) $a+b+c$ (d) $9x^2+a+b+c$

27. The local minimum value of the function f' given by $f(x)=3+|x|, x\in R$ is

(a) -1 (b) 3 (c) 1 (d) 0

28. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/sec. At that instant, when the radius of circular wave is 8 cm, how fast is the enclosed area increasing?

(a) 6π cm^2/s (b) 8π cm^2/s

(c) $\frac{8}{3}$ cm^2/s (d) 80π cm^2/s

29. A gardener is digging a plot of land. As he gets tired, he works more slowly. After 't' minutes he is digging at a rate of $\frac{2}{\sqrt{t}}$ square metres per minute. How long will it take him to dig an area of 40 square metres?

(a) 100 minutes (b) 10 minutes

(c) 30 minutes (d) 40 minutes

30. The area of the region bounded by the lines $y=mx$, $x=1$, $x=2$, and x axis is 6 sq. units, then 'm' is

(a) 3 (b) 1 (c) 2 (d) 4

31. Area of the region bounded by two parabolas $y=x^2$ and $x=y^2$ is

(1) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) 4 (d) 3

32. The order and degree of the differential equation $y=x\frac{dy}{dx}+\frac{2}{\frac{dy}{dx}}$ is

(a) 1, 2 (b) 1, 3 (c) 2, 1 (d) 1, 1

33. The general solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=3x$ is

(a) $y=x-\frac{c}{x}$ (b) $y=x+\frac{c}{x}$

(c) $y=x^2-\frac{c}{x}$ (d) $y=x^2+\frac{c}{x}$

34. The distance of the point $P(a, b, c)$ from the x-axis is

(a) $\sqrt{a^2+b^2}$ (b) $\sqrt{b^2+c^2}$

(c) a (d) $\sqrt{a^2+c^2}$

35. Equation of the plane perpendicular to the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and passing through the point (2, 3, 4) is

(a) $2x+3y+z=17$ (b) $x+2y+3z=9$

(c) $3x+2y+z=16$ (d) $x+2y+3z=20$

36. The line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is parallel to the plane

(a) $2x+3y+4z=0$ (b) $3x+4y+5z=7$

(c) $2x+y-2z=0$ (d) $x+y+z=2$

37. The angle between two diagonals of a cube is

(a) $\cos^{-1}\left(\frac{1}{3}\right)$ (b) 30°

(c) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) 45°

38. Lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-K}$ and $\frac{x-1}{K}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

(a) $K=2$ (b) $K=0$ (c) $K=3$ (d) $K=-1$

39. A and B are two events such that $P(A) \neq 0$, $P(B/A)$ if

(i) A is a subset of B

(ii) $A \cap B = \phi$ are respectively

(a) 1, 1 (b) 0 and 1

(c) 0, 0 (d) 1, 0

40. Two dice are thrown simultaneously. The probability of obtaining a total score of 5 is

(a) $\frac{1}{9}$ (b) $\frac{1}{18}$ (c) $\frac{1}{36}$ (d) $\frac{1}{12}$

41. If the events A and B are independent if $P(A')=\frac{2}{3}$ and $P(B')=\frac{2}{7}$, then $P(A \cap B)$ is equal to

(a) $\frac{4}{21}$ (b) $\frac{5}{21}$ (c) $\frac{1}{21}$ (d) $\frac{3}{21}$

42. A box contains 100 bulbs, out of which 10 are defective. A sample of 5 bulbs is drawn. The probability that none is defective is

(a) $\frac{9}{10}$ (b) $\left(\frac{1}{10}\right)^5$ (c) $\left(\frac{9}{10}\right)^5$ (d) $\left(\frac{1}{2}\right)^5$

43. The area of the parallelogram whose adjacent sides are $\hat{i}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ is

(a) 3 (b) $\sqrt{2}$ (c) 4 (d) $\sqrt{3}$

44. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\frac{\pi}{3}$, then the value of $|\vec{a}+\vec{b}|$ is

(a) equal to 1 (b) greater than 1

(b) equal to 0 (c) less than 1

45. The value of $\left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}\right]$ is equal to

(a) 0 (b) 1

(c) $2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ (d) 2

46. If $x + y \leq 2$, $x \geq 0$, $y \geq 0$ the point at which maximum value of $3x + 2y$ attained will be

(a) (0, 2) (b) (0, 0) (c) (2, 0) (d) $\left(\frac{1}{2}, \frac{1}{2}\right)$

47. If $\sin\theta = \sin\alpha$, then

(a) $\frac{\theta+\alpha}{2}$ is any multiple of $\frac{\pi}{2}$ and $\frac{\theta-\alpha}{2}$ is any odd multiple of π.

(b) $\frac{\theta+\alpha}{2}$ is any odd multiple of $\frac{\pi}{2}$ and $\frac{\theta-\alpha}{2}$ is any multiple of π.

(c) $\frac{\theta+\alpha}{2}$ is any multiple of $\frac{\pi}{2}$ and $\frac{\theta-\alpha}{2}$ is any even multiple of π.

(d) $\frac{\theta+\alpha}{2}$ is any even multiple of $\frac{\pi}{2}$ and $\frac{\theta-\alpha}{2}$ is any odd multiple of π.

48. If $\tan x=\frac{3}{4}, \pi < x < \frac{3\pi}{2}$, then the value of $\cos\frac{x}{2}$ is

(a) $-\frac{1}{\sqrt{10}}$ (b) $\frac{3}{\sqrt{10}}$ (c) $\frac{1}{\sqrt{10}}$ (d) $-\frac{3}{\sqrt{10}}$

49. In a triangle ABC, $a[b\cos C - c\cos B] =$

(a) 0 (b) a^2

(c) $b^2 - c^2$ (d) b^2

50. If α and β are two different complex numbers with $|\beta|=1$, then $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$ is equal to

(a) $\frac{1}{2}$ (b) 0 (c) –1 (d) 1

51. The set $A = \{x : |2x + 3| < 7\}$ is equal to the set

(a) $D = \{x : 0 < x + 5 < 7\}$

(b) $B = \{x : -3 < x < 7\}$

(c) $E = \{x : -7 < x < 7\}$

(d) $C = \{x : -13 < 2x < 4\}$

52. How many 5 digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once?

(a) 335 (b) 336 (c) 338 (d) 337

53. If 21st and 22nd terms in the expansion of $(1 + x)^{44}$ are equal, then x is equal to

(a) $\frac{8}{7}$ (b) $\frac{21}{22}$ (c) $\frac{7}{8}$ (d) $\frac{23}{24}$

54. Consider an infinite geometric series with first term 'a' and common ratio 'r'. If the sum is

4 and the second term is $\frac{3}{4}$, then

(a) $a=2, r=\frac{3}{8}$ (b) $a=\frac{4}{7}, r=\frac{3}{7}$

(c) $a=\frac{3}{2}, r=\frac{1}{2}$ (d) $a=3, r=\frac{1}{4}$

55. A straight line passes through the points (5, 0) and (0, 3). The length of perpendicular from the point (4, 4) on the line is

(a) $\frac{15}{\sqrt{34}}$ (b) $\frac{\sqrt{17}}{2}$ (c) $\frac{17}{2}$ (d) $\sqrt{\frac{17}{2}}$

56. Equation of circle with centre $(-a, -b)$ and radius $\sqrt{a^2-b^2}$ is

(a) $x^2+y^2+2ax+2by+2b^2=0$

(b) $x^2+y^2-2ax-2by-2b^2=0$

(c) $x^2+y^2-2ax-2by+2b^2=0$

(d) $x^2+y^2-2ax+2by+2a^2=0$

57. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of latus rectum is

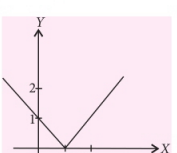
(a) 20 sq. units (b) 18 sq. units

(c) 17 sq. units (d) 19 sq. units

58. If the coefficient of variation and standard deviation are 60 and 21 respectively, the arithmetic mean of distribution is

(a) 60 (b) 30 (c) 35 (d) 21

59. The function represented by the following graph is



(a) Continuous but not differentiable at $x = 1$

(b) Differentiable but not continuous at $x = 1$

(c) Continuous and differentiable at $x = 1$

(d) Neither continuous nor differentiable at $x = 1$

60. If $f(x)=\begin{cases}\frac{3\sin \pi x}{5x}, & x\neq 0\\ 2K, & x=0\end{cases}$ is continuous at $x = 0$, then the value of K is

(a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{10}$ (b) $\frac{3\pi}{2}$ (d) $\frac{3\pi}{5}$

SOLUTIONS

1. (c)

2. (b) : We have,

$y=(1+x)(1+x^2)(1+x^4)$

$=1+x+x^2+x^3+x^4+x^5+x^6+x^7$

On differentiating, we get

$\frac{dy}{dx}=1+2x+3x^2+4x^3+5x^4+6x^5+7x^6$

$\therefore \left.\frac{dy}{dx}\right|_{\text{at } x=1} = 1+2+3+4+5+6+7=28$

3. (c): We have, $y=(\tan^{-1}x)^2$

On differentiating, we get

$$y_1=\frac{2\tan^{-1}x}{1+x^2} \quad ...(1)$$

Again differentiating (1), we get

$$y_2=2\left\{\frac{(1+x^2)\times\frac{1}{1+x^2}-\tan^{-1}x\times 2x}{(1+x^2)^2}\right\}$$

$$=2\left\{\frac{1-2x\tan^{-1}x}{(1+x^2)^2}\right\} \quad ...(2)$$

Now, using (1) and (2), we have

$(x^2+1)^2y_2+2x(x^2+1)y_1$

$=(x^2+1)^2\left[2\left(\frac{1-2x\tan^{-1}x}{(1+x^2)^2}\right)\right]$

$+2x(x^2+1)\left[\frac{2\tan^{-1}x}{1+x^2}\right]$

$=2-4x\tan^{-1}x+4x\tan^{-1}x=2$

4. (d) : We have, $f(x)=x^3$

We observe that,

(i) $f(x)$ being a polynomial function, is continuous on $[-2, 2]$

(ii) $f'(x)=3x^2$, which clearly exists for all values of $x\in]-2, 2[$

(iii) $f(-2)=-8, f(2)=8$

since, $f(-2)\neq f(2)$

Therefore, Rolle's theorem is not satisfied.

(iv) Since $f(x)$ is continuous and differentiable, so there exists some $c \in]-2, 2[$, such that

$$f'(c) = \frac{f(2)-f(-2)}{(2+2)} = \frac{8+8}{4} = \frac{16}{4} = 4$$

Now, $f'(c) = 4 \Leftrightarrow 3c^2 = 4 \Leftrightarrow c = \pm\frac{2}{\sqrt{3}} \in]-2, 2[$

Hence, mean value theorem is satisfied.

Again, we have, $g(x) = x^3 - 4x$

We observe that,

(i) $g(x)$ being a polynomial function, is continuous on $[-2, 2]$

(ii) $g'(x) = 3x^2 - 4$, which exists for all values of $x \in]-2, 2[$

(iii) $g(-2) = -8 + 8 = 0, g(2) = 8 - 8 = 0$

$\therefore \quad g(-2) = g(2)$

Thus, all conditions of Rolle's theorem are satisfied. So, there exists a real number $c \in]-2, 2[$, such that

$$g'(c) = 0$$

Now, $g'(c) = 0 \Leftrightarrow 3c^2 - 4 = 0$

$$\Leftrightarrow \quad c = \pm\frac{2}{\sqrt{3}} \in]-2, 2[$$

Hence, Rolle's theorem is satisfied.

(iv) We know that, every function that satisfies Rolle's theorem will also satisfy the mean value theorem.

Hence, $f(x)$ and $g(x)$ satisfy mean value theorem and only $g(x)$ satisfy Rolle's theorem.

5. (a)

6. (b) : We have, $\lim_{x\to 1} \frac{f(x)-2}{x^2-1} = \pi$

$$\Rightarrow \quad \lim_{x\to 1}[f(x)-2] = \pi \lim_{x\to 1}(x^2-1) = 0$$

$$\Rightarrow \quad \lim_{x\to 1} f(x) = 2$$

7. (d) : The given curve is, $y = x^3 + 1$

$$\therefore \quad \frac{dy}{dx} = 3x^2 \Rightarrow \left.\frac{dy}{dx}\right|_{\text{at }(1,2)} = 3$$

Now, $\tan\theta = \frac{dx}{dy} = \frac{1}{3}$

8. (c): We have, $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1$

$$\Rightarrow \quad f'(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + ... + \frac{2x}{2} + 1$$

$$= x^{99} + x^{98} + ...+ x + 1$$

$\therefore \quad f'(0) = 0 + 0 + ...+ 0 + 1 = 1$

9. (c): Let $I = \int_e^{\pi} xf(x)dx = \int_e^{\pi} (\pi + e - x) f(\pi + e - x)dx$

$$\Rightarrow \quad I = \int_e^{\pi} (\pi + e - x) f(x)dx \quad [\because f(x) = f(\pi + e - x)]$$

$$\Rightarrow \quad I = \int_e^{\pi e} (\pi + e) f(x)dx - \int_e^{\pi} xf(x)dx$$

$$= (\pi + e)\int_e^{\pi} f(x)dx - I$$

$$\Rightarrow \quad 2I = (\pi + e) \times \frac{2}{\pi + e} \Rightarrow 2I = 2 \Rightarrow I = 1$$

$$\therefore \quad \int_e^{\pi} xf(x)dx = 1$$

10. (c): Now, $\int (3x\cos x - \cos x + \sin x - 2x\sin x)dx$

$$= f(x)\cos x + g(x)\sin x + C$$

$$\Rightarrow \quad 3\int x\cos x dx - \sin x - \cos x - 2\int x\sin x dx$$

$$= f(x)\cos x + g(x)\sin x + C$$

$$\Rightarrow \quad 3[x \cdot \sin x - \int 1 \cdot \sin x dx] - \sin x - \cos x$$

$$-2[x \cdot (-\cos x) - \int 1 \cdot (-\cos x)dx]$$

$$= f(x)\cos x + g(x)\sin x + C$$

$$\Rightarrow \quad 3x\sin x + 3\cos x - \sin x - \cos x + 2x\cos x - 2\sin x + C$$

$$= f(x)\cos x + g(x)\sin x + C$$

$$\Rightarrow \quad 2(x+1)\cos x + 3(x-1)\sin x + C$$

$$= f(x)\cos x + g(x)\sin x + C$$

$$\Rightarrow \quad f(x) = 2(x+1),\ g(x) = 3(x-1)$$

11. (a) : Let $I = \int_{-\pi/4}^{\pi/4} \log(\sec\theta - \tan\theta)d\theta$

$$I = \int_{-\pi/4}^{\pi/4} \log(1 - \sin\theta)d\theta - \int_{-\pi/4}^{\pi/4} \log\cos\theta d\theta \qquad ...(1)$$

$$I = \int_{-\pi/4}^{\pi/4} \log\left(1 - \sin\left(\frac{\pi}{4} - \frac{\pi}{4} - \theta\right)\right)d\theta$$

$$- \int_{-\pi/4}^{\pi/4} \log\cos\left(\frac{\pi}{4} - \frac{\pi}{4} - \theta\right)d\theta$$

$$= \int_{-\pi/4}^{\pi/4} \log(1 + \sin\theta)d\theta - \int_{-\pi/4}^{\pi/4} \log\cos\theta\, d\theta \qquad ...(2)$$

Adding (1) and (2), we get

$$2I=\int_{-\pi/4}^{\pi/4}\log\left(1-\sin^2\theta\right)d\theta-\int_{-\pi/4}^{\pi/4}\log(\cos^2\theta)d\theta=0$$

12. (c): Let $I=\int\frac{\sin 2x}{\sin^2 x+2\cos^2 x}dx$

$$=\int\frac{2\sin x\cos x}{1+\cos^2 x}dx$$

Put $1+\cos^2 x=t \Rightarrow -2\sin x\cos x\,dx=dt$

$\therefore \quad I=-\int\frac{dt}{t}=-\log t+C=-\log(1+\cos^2 x)+C$

13. (c): For Reflexive : Let $a \in S$.

Then, $|a-a|=0<1 \Rightarrow aRa$.

So, R is reflexive.

For symmetric : Let $aRb \Leftrightarrow |a-b|\le 1$

$\Leftrightarrow |-(b-a)|\le 1$

$\Leftrightarrow |b-a|\le 1 \Leftrightarrow bRa$.

So, R is symmetric

For Transitive :

Let, $1, 2, 3 \in S$

Then, $|1-2|=1\le 1 \Rightarrow 1R2$

and $|2-3|=1\le 1 \Rightarrow 2R3$

But $|1-3|=2>1$

So, R is not transitive.

14. (b) : We have $a*b=1+ab$

Now, $a*b=1+ab=1+ba=b*a$

So, * is commutative.

Again, $a*(b*c)=1+a(b*c)=1+a(1+bc)$

$=1+a+abc$

and, $(a*b)*c=1+(a*b)c=1+(1+ab)c$

$=1+c+abc$

$\therefore$ * is not associative.

15. (a) : We have, $f(1)=\frac{(1+1)}{2}=\frac{2}{2}=1$ and $f(2)=\frac{2}{2}=1$

Thus, $f(1)=f(2)$ but $1\ne 2$

$\therefore$ $f(n)$ is not one-one.

In order to find that f is onto or not, consider an arbitrary element $n \in N$.

If n is odd, then $(2n-1)$ is also odd and

$$f(2n-1)=\frac{(2n-1+1)}{2}=\frac{2n}{2}=n$$

If n is even, then $2n$ is also even and

$$f(2n)=\frac{2n}{2}=n$$

Thus, for each $n \in N$, there exists its pre-image in N.

$\therefore$ f is onto.

16. (d)

17. (d) : We have, $f(x)=\sqrt{\cos x}$

$\sqrt{\cos x}$ is defined when cos$x$ is positive or zero, *i.e.*, 1st or 4th quadrant

So, domain $=\left[0,\frac{\pi}{2}\right]\cup\left[\frac{3\pi}{2},2\pi\right]$

18. (c): Total number of students, $(N)=60$

Let $n(C)$ be the number of students who play cricket and $n(T)$ be the number of students who play tennis.

$\therefore$ $n(C)=25$, $n(T)=20$, $n(C\cap T)=10$

So, $n(C\cup T)=n(C)+n(T)-n(T\cap C)$

$=25+20-10=35$

Now, number of students who play neither game $=n(C\cup T)'=N-n(C\cup T)=60-35=25$

19. (a) : Let,

$$y=\tan\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right\}-\sin^{-1}x\right]$$

Put $x=\sin\theta$, we get

$$y=\tan\left[\sin^{-1}\left[\frac{\sin\theta}{\sqrt{2}}+\frac{\sqrt{1-\sin^2\theta}}{\sqrt{2}}\right]-\sin^{-1}(\sin\theta)\right]$$

$$=\tan\left[\sin^{-1}\left[\frac{\sin\theta+\cos\theta}{\sqrt{2}}\right]-\theta\right]$$

$$=\tan\left[\sin^{-1}\left\{\sin\theta\cos\frac{\pi}{4}+\cos\theta\sin\frac{\pi}{4}\right\}-\theta\right]$$

$$=\tan\left[\sin^{-1}\left\{\sin\left(\theta+\frac{\pi}{4}\right)\right\}-\theta\right]$$

$$=\tan\left[\theta+\frac{\pi}{4}-\theta\right]=\tan\frac{\pi}{4}=1$$

20. (d) : We have,

$$\sin[2\sin^{-1}0.8]=\sin\left[\sin^{-1}\left(2\times 0.8\sqrt{1-(0.8)^2}\right)\right]$$

$$=2\times 0.8\times 0.6=0.96$$

21. (b)

22. (d) : $A=\begin{bmatrix}1 & 2 & 4\\ 6 & 8 & 2\\ 2 & -2 & 7\end{bmatrix}$

$$A' = \begin{bmatrix} 1 & 6 & 2 \\ 2 & 8 & -2 \\ 4 & 2 & 7 \end{bmatrix}$$

Now, symmetric part of $A = \frac{1}{2}(A + A')$

$$= \frac{1}{2}\begin{bmatrix} 1 & 2 & 4 \\ 6 & 8 & 2 \\ 2 & -2 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 & 2 \\ 2 & 8 & -2 \\ 4 & 2 & 7 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 8 & 6 \\ 8 & 16 & 0 \\ 6 & 0 & 14 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 \\ 4 & 8 & 0 \\ 3 & 0 & 7 \end{bmatrix}$$

23. (c): We know that, $A \cdot (\text{adj } A) = |A| \cdot I$
and it is given that, $A \cdot (\text{adj } A) = 10 \cdot I$
$\therefore \quad |A| = 10$
Also, $|\text{adj } A| = |A|^{n-1}$
$\therefore \quad |\text{adj } A| = (10)^2 = 100$

24. (c)

25. (d) : $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{vmatrix} = 2(12 - 0) - 0 + 0 = 24 \neq 0$

Thus, $|A| \neq 0$, so A^{-1} exists.

Now, cofactor matrix of $A = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$$\therefore \quad (\text{Adj } A) = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$$= \frac{1}{24}\begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

26. (a) : It is given that, a, b, c are in A.P.
$\therefore \quad a + c = 2b$ ... (1)
Now,

$$\begin{bmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{bmatrix}$$

Applying $R_1 \to R_1 + R_3$, we get

$$= \begin{bmatrix} 2x+8 & 2x+10 & 2x+(a+c) \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{bmatrix}$$

$$= \begin{bmatrix} 2(x+4) & 2(x+5) & 2x+2b \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{bmatrix} \quad \{\text{Using (1)}\}$$

$$= 2\begin{bmatrix} x+4 & x+5 & x+b \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{bmatrix} = 0$$

{$\because$ If any two rows (or columns) of a determinant are identical, then the value of the determinant is 0}.

27. (a, c): $f'(x) = x, x > 0$ and $f'(x) = -1, x < 0; f'(0)$ doesn't exist
Obviously $f'(x) = -1$ is a local minimum value.
$\therefore$ At every +ve real number, 1 is the local maximum as well as local minimum.
$\therefore$ 1 is also real minimum value.

28. (d) : Let 'r' be the radius of the circular wave.
It is given that, $\frac{dr}{dt} = 5$ cm/sec and $r = 8$ cm
Now, Area $(A) = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

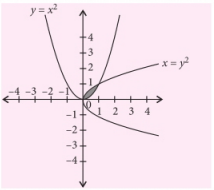
$\Rightarrow \frac{dA}{dt} = 2 \times \pi \times 8 \times 5 = 80\ \pi\ \text{cm}^2/\text{sec}.$

29. (a) : If x is the area of the area dug at any time t, then $\frac{dx}{dt} = \frac{2}{\sqrt{t}} \quad \therefore \ x = 4\sqrt{t} + c$
When $t = 0, x = 0 \quad \therefore \ c = 0 \ \therefore \ x = 4\sqrt{t}$
When $x = 40, 40 = 4\sqrt{t} \ \Rightarrow \ \sqrt{t} = 10 \ \therefore \ t = 100$ min.

30. (d) : Given area $= \int_1^2 mx\ dx$

$\Rightarrow \quad 6 = m\left[\frac{x^2}{2}\right]_1^2 \Rightarrow \quad 6 = m\left[\frac{4}{2} - \frac{1}{2}\right] \Rightarrow m = 4$

31. (b) :



Required area $= \int_0^1 \sqrt{x}dx - \int_0^1 x^2 dx$

$= \left|\frac{2x^{3/2}}{3}\right|_0^1 - \left|\frac{x^3}{3}\right|_0^1 = \frac{2}{3}[1-0] - \frac{1}{3}[1-0]$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ sq. units

32. (a) : The given differential equation becomes,

$y\left(\frac{dy}{dx}\right) = x\left(\frac{dy}{dx}\right)^2 + 2$

So, order = 1, degree = 2

33. (c,d) : The given differential equation is,

$\frac{dy}{dx} + \frac{y}{x} = 3x$ ...(1)

It is a linear differential equation.

So, I.F. $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$

$\therefore$ Solution of eq. (1) is given by

$x \cdot y = \int 3x^2 dx$

$\Rightarrow \quad xy = 3\left[\frac{x^3}{3}\right] + c \quad \text{or} \quad xy + c = 3\left[\frac{x^3}{3}\right]$

$\Rightarrow \quad xy = x^3 + c$ or $xy + c = x^3$

$\Rightarrow \quad y = x^2 + \frac{c}{x} \quad \text{or} \quad y = x^2 - \frac{c}{x}$

34. (b)

35. (d) : Let the equation of plane be,

$ax + by + cz + d = 0$ ... (1)

Now, the given line is $\frac{x-0}{1} = \frac{y-0}{2} = \frac{z-0}{3}$ ... (2)

Also, plane (1) is perpendicular to the line (2).

So, $\frac{a}{1} = \frac{b}{2} = \frac{c}{3} = \lambda$ (say)

$\Rightarrow \quad a = \lambda,\ b = 2\lambda,\ c = 3\lambda$ ... (3)

Also, plane (1) passes through (2, 3, 4). So, we get

$2a + 3b + 4c + d = 0$ ... (4)

From (3) and (4), we have

$2\lambda + 6\lambda + 12\lambda + d = 0 \ \Rightarrow \ d = -20\lambda$

$\therefore$ Required equation of plane is,

$\lambda x + 2\lambda y + 3\lambda z - 20\lambda = 0 \ \Rightarrow \ x + 2y + 3z = 20$

36. (c) : Let the equation of plane be

$ax + by + cz + d = 0$ ... (1)

The given line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is parallel to plane (1).

So, $3a + 4b + 5c = 0$

Now, from options only plane in (c) satisfied the equation. So, (c) is the correct option.

37. (a)

38. (b) : The given lines are,

$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-K}$ ...(1)

and $\frac{x-1}{K} = \frac{y-4}{2} = \frac{z-5}{1}$ ... (2)

Now, (1) and (2) are coplanar.

$\Rightarrow \quad \begin{vmatrix} 1-2 & 4-3 & 5-4 \\ 1 & 1 & -K \\ K & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -K \\ K & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow \ -1(1 + 2K) - 1(1 + K^2) + 1(2 - K) = 0$

$\Rightarrow \ K^2 + 3K = 0 \ \Rightarrow \ K(K + 3) = 0$

$\Rightarrow K = 0$ or $K = -3$

39. (d) : (i) It is given that, $A \subset B$

$\Rightarrow \ A \cap B = A$

$\therefore \quad P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1$

(ii) If $A \cap B = \phi$

Then $P(B/A) = \frac{P(A \cap B)}{P(A)} = 0$

40. (a) : Total number of possible outcomes = 36

Let E be the event of getting a total of 5.
So, $E = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$
$\therefore \quad n(E) = 4$

So, required probability $P(E) = \frac{4}{36} = \frac{1}{9}$

41. (b) : It is given that,

$$P(A') = \frac{2}{3} \Rightarrow P(A) = 1 - P(A') = 1 - \frac{2}{3} = \frac{1}{3}$$

Also, $P(B') = \frac{2}{7} \Rightarrow P(B) = 1 - \frac{2}{7} = \frac{5}{7}$

Now, A and B are independent events.

So, $P(A \cap B) = P(A) \cdot P(B) = \frac{1}{3} \times \frac{5}{7} = \frac{5}{21}$

42. (c): Total number of bulbs = 100

Probability of defective bulbs $(p) = \frac{10}{100} = \frac{1}{10}$

$\therefore$ Probability of non-defective bulbs$(q) = \frac{9}{10}$

So, required probability, $P(X = 0) = {}^nC_0 (p)^0 (q)^{n-0}$

$$= {}^5C_0 \times \left(\frac{9}{10}\right)^5 = \left(\frac{9}{10}\right)^5$$

43. (d) : Let $\vec{a} = \hat{i} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix}$

$= \hat{i}(0-1) - \hat{j}(1-2) + \hat{k}(1-0) = -\hat{i} + \hat{j} + \hat{k}$

Area $= |\vec{a} \times \vec{b}| = \left|-\hat{i} + \hat{j} + \hat{k}\right| = \sqrt{1+1+1} = \sqrt{3}$ sq. units

44. (b) : Since $\vec{a}$ and $\vec{b}$ are two unit vectors.

$\therefore \quad |\vec{a}| = |\vec{b}| = 1$

Now, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta = 1 \cdot 1 \cdot \cos\frac{\pi}{3} = \frac{1}{2}$

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a} \cdot \vec{b}) = 1 + 1 + 1 = 3$

$\Rightarrow \quad |\vec{a} + \vec{b}| = \sqrt{3} > 1$

45. (a) : We have, $\left[\vec{a} - \vec{b} \quad \vec{b} - \vec{c} \quad \vec{c} - \vec{a}\right]$

$= (\vec{a} - \vec{b}) \cdot \{(\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})\}$

$= (\vec{a} - \vec{b}) \cdot \{\vec{b} \times \vec{c} - \vec{b} \times \vec{a} + \vec{c} \times \vec{a}\} \quad \left[\because \ \vec{c} \times \vec{c} = 0\right]$

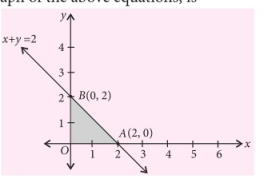
$= \vec{a} \cdot \vec{b} \times \vec{c} - \vec{b} \cdot \vec{c} \times \vec{a}$

$= [\vec{a}\ \vec{b}\ \vec{c}] - [\vec{a}\ \vec{b}\ \vec{c}] = 0$

46. (c): Let $Z = 3x + 2y$
So, we have

max.	$Z = 3x + 2y$
such that	$x + y \le 2$
	$x \ge 0,\ y \ge 0$

Convert the given inequations into equations.
So, we have, $x + y = 2$, $x = 0$, $y = 0$
The graph of the above equations, is



Experts crack Ramanujan's algebra riddle

Mathematicians have found a framework for the celebrated Rogers-Ramanujan identities and their arithmetic properties, solving another long-standing mystery stemming from the work of Indian math genius Srinivasa Ramanujan. The solution was found by mathematicians at Emory University and the University of Queensland. "Algebraic numbers are among the first numbers you encounter in mathematics and yet it's difficult to find functions that return them as values in a uniform and systematic way," said Ken Ono, a number theorist at Emory. "A fundamental problem in mathematics is to find functions whose values are always algebraic numbers," he said. Ramanujan could produce such numbers, and he made it look easy. The Rogers-Ramanujan identities are considered among his greatest legacies. The identities were originally discovered by L J Rogers in 1894. In 1913, Ramanujan sent a letter to British mathematician G H Hardy that included the two identities that Rogers discovered and a third formula that showed these identities are essentially modular functions and their quotient has the special property that its singular values are algebraic integral units. That result came to be known as the Rogers-Ramanujan continued fraction. Ramanujan died in 1920 before he could explain how he conjured up the formulas. For nearly a century, experts have tried to solve the mystery.

Courtesy : TOI

So, the feasible region is shaded in the graph.

$\therefore$ Points are $A(2, 0)$, $B(0, 2)$ and $O(0, 0)$

Value of Z at $A = 3 \times 2 + 2 \times 0 = 6$

Value of Z at $B = 3 \times 0 + 2 \times 2 = 4$

Value of Z at $O = 3 \times 0 + 2 \times 0 = 0$

$\therefore$ Maximum value of $Z = 6$ which is at $(2, 0)$

47. $\sin\theta = \sin\alpha \Rightarrow \sin\theta - \sin\alpha = 0$

$$\therefore \quad 2\cos\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right)=0$$

$$\therefore \quad \cos\left(\frac{\theta+\alpha}{2}\right)=0 \text{ or } \sin\left(\frac{\theta-\alpha}{2}\right)=0$$

$\therefore \quad \dfrac{\theta+\alpha}{2}$ is multiple of $\dfrac{\pi}{2}$ or $\dfrac{\theta-\alpha}{2}$ is a multiple of π

$\therefore$ None of the given options is correct.

Note : Option (b) would have been the correct answer if 'and' is replaced by 'or'.

48. (a) : Since x lies in 3rd quadrant.

So, $\cos x$ and $\cos\dfrac{x}{2}$ are negative

Now, $\tan x = \dfrac{3}{4} \Rightarrow \cos x = -\dfrac{4}{5}$

Also, $\cos x = 2\cos^2\dfrac{x}{2} - 1$

$$\Rightarrow \quad \cos^2\frac{x}{2}=\frac{1}{2}\{\cos x+1\}=\frac{1}{2}\left\{-\frac{4}{5}+1\right\}=\frac{1}{10}$$

$$\Rightarrow \quad \cos\frac{x}{2}=\frac{-1}{\sqrt{10}}$$

49. (c): We have,

$a[b\cos C - c\cos B] = ab\cos C - ac\cos B$

$$= ab\times\frac{a^2+b^2-c^2}{2ab}-ac\times\frac{a^2+c^2-b^2}{2ac}$$

$$=\frac{(a^2+b^2-c^2)-(a^2+c^2-b^2)}{2}$$

$$=\frac{2b^2-2c^2}{2}=b^2-c^2$$

50. (d) :

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=\frac{\sqrt{(\beta-\alpha)(\bar{\beta}-\bar{\alpha})}}{\sqrt{(1-\bar{\alpha}\beta)(1-\bar{\alpha}\bar{\beta})}}=\frac{\sqrt{(\beta-\alpha)(\bar{\beta}-\bar{\alpha})}}{\sqrt{(1-\bar{\alpha}\beta)(1-\bar{\alpha}\bar{\beta})}}$$

$$=\sqrt{\frac{\beta\bar{\beta}-\beta\bar{\alpha}-\alpha\bar{\beta}+\alpha\bar{\alpha}}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+\alpha\bar{\alpha}\beta\bar{\beta}}}=\sqrt{\frac{|\beta|^2-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2|\beta|^2}}$$

$$=\sqrt{\frac{1-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}}=\sqrt{1}=1 \qquad (\because\ |\beta|=1)$$

51. (a) : We have, $|2x+3| < 7$

$\Rightarrow \quad -7 < 2x + 3 < 7 \Rightarrow -7 - 3 < 2x < 7 - 3$

$\Rightarrow \quad -10 < 2x < 4 \Rightarrow -5 < x < 2$

$\Rightarrow \quad 0 < x + 5 < 7$

52. (b) : Required number $= 1 \times 1 \times {}^8P_3 = 336$

53. (c): $T_{21} = T_{20+1} = {}^{44}C_{20}\,x^{20}$

$T_{22} = T_{21+1} = {}^{44}C_{21}\,x^{21}$

According to question, ${}^{44}C_{20}\,x^{20} = {}^{44}C_{21}\,x^{21}$

$$\Rightarrow \quad x=\frac{{}^{44}C_{20}}{{}^{44}C_{21}}\Rightarrow x=\frac{7}{8}$$

54. (d) : We have, $ar = \dfrac{3}{4}$ ... (1)

and $S_\infty = 4 \Rightarrow \dfrac{a}{1-r} = 4$

$$\Rightarrow \quad \frac{a}{1-\frac{3}{4a}}=4 \qquad \{\text{Using (1)}\}$$

$\Rightarrow \quad a^2 - 4a + 3 = 0 \quad \Rightarrow (a - 3)(a - 1) = 0$

$\Rightarrow \quad a = 3$ or $a = 1$

When $a = 3$, $r = \dfrac{1}{4}$ or when $a = 1$, $r = \dfrac{3}{4}$

55. (d) : Equation of line passing through (5, 0) and (0, 3) is, $y-3=\dfrac{-3}{5}(x-0)$

$\Rightarrow \quad 5y - 15 = -3x$

$\Rightarrow \quad 3x + 5y - 15 = 0$ ... (1)

Now, perpendicular distance

$$=\left|\frac{4\times3+4\times5-15}{\sqrt{9+25}}\right|=\left|\frac{12+20-15}{\sqrt{34}}\right|=\left|\frac{17}{\sqrt{34}}\right|=\sqrt{\frac{17}{2}}$$

56. (a) : Equation of circle is

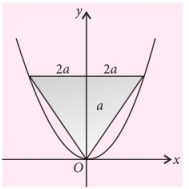
$$(x+a)^2+(y+b)^2=\left(\sqrt{a^2-b^2}\right)^2$$

$\Rightarrow \quad x^2 + a^2 + 2ax + y^2 + b^2 + 2by = a^2 - b^2$

$\Rightarrow \quad x^2 + y^2 + 2ax + 2by + 2b^2 = 0$

57. (b) : Area of triangle $= \dfrac{1}{2}(4a)(a) = 2a^2$

$= 2 \cdot 3^2 = 18$ sq. units



58. (c): We know that,

$$\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100 \Rightarrow 60 = \frac{21}{\bar{x}} \times 100 \Rightarrow \bar{x} = 35$$

59. (a) : From the given graph, the function is continuous.

The given graph is that of $y = |x - 1|$

Now, $f(x) = y = \begin{cases} x-1, & x \geq 1 \\ 1-x, & x < 1 \end{cases}$

$$Rf'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h-1) - 0}{h} = 1$$

$$Lf'(1) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \to 0} \frac{(1-1+h) - 0}{-h} = -1$$

Since $Rf'(1) \neq Lf'(1)$

So, the given function is not differentiable.

60. (b) : Clearly, $f(0) = 2K$

Since, the given function is continuous

$$\lim_{x \to 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x \to 0} \frac{3 \sin \pi x}{5x} = 2K$$

$$\Rightarrow \frac{3}{5} \lim_{x \to 0} \frac{\sin \pi x}{\pi x} \times \pi = 2K$$

$$\Rightarrow \frac{3}{5}\pi = 2\ K \Rightarrow K = \frac{3\pi}{10}$$

HOW BRAIN INJURY TURNED MAN INTO A MATHS GENIUS

Scientists have made some progress in figuring out how a man who received severe brain injuries suddenly became a mathematical genius. They say that an area behind the crown of the head, known as the parietal cortex, appears to have become more active, according to a report in Live Science. This region is known to combine information from different senses. Jason Padgett was an ordinary furniture salesman in Tacoma, Washington, US. In 2002, he was assaulted by two men outside a karaoke bar resulting in severe concussion and an injured kidney. As Padgett recovered, he suffered from post-traumatic stress disorder. As he progressed, Padgett realized that he was seeing the world differently - everything looked like it was made up of geometrical shapes. He saw a circle as made up of overlapping triangles. He could draw complex geometric shapes. He saw shapes when shown mathematical equations. One day a physicist saw him making these shapes in a mall and was struck by Padgett's abilities. He persuaded Padgett to join college, where he is studying number theory. As his abilities and how he acquired them got known, brain scientists got interested in finding out what had happened in his brain. Berit Brogaard, a philosophy professor now at the University of Miami and her colleagues used functional magnetic resonance imaging (fMRI) to study Padgett's brain. The scans showed that the left parietal cortex lit up the most, while areas involved with visual memory, sensory processing and planning also showed activity. Using transcranial magnetic stimulation the scientists zapped specific areas with a magnetic pulse which either activates or inhibits the area. When the parietal cortex was thus zapped, the synesthesia faded. According to Live Science, Brogaard has earlier shown that when brain cells die, they release chemicals to increase activity in surrounding areas. This may have happened in Padgett's case. It appears that abilities like Padgett's may be dormant in every brain and they got released after the injury.

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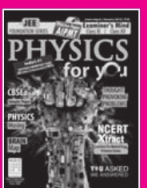

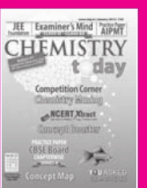

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